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NATURAL PHILOSOPHY.

PART I.

MECHANICS.

BY

J. ALFRED SKERTCHLY.

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PREFACE.

THIS work is mainly designed for that class of students whose mathematical knowledge is not sufficiently advanced to enable them to enter into the higher branches and proofs of mechanical science. Each theorem is explained in the text in simple language, while at the same time the chief algebraical formulæ are added in the shape of foot-notes, so as to be available if required.

The method of dividing the subject is that adopted by Professors Thomson and Tait, in which Mechanics falls under the two heads Kinematics and Dynamics, a mode of division which, though novel, is likely to be the one that will ultimately prevail.

In Kinematics, motion in the abstract is first discussed. Then follows a very original and lucid chapter on the Graphic Representation of Motion, contributed by F. J. Rudler, Esq., of the Royal School of Mines.

In the section on Dynamics, Statics, or forces in equilibrium, is first considered, including the mechanical powers, the modifications of motion used in machinery, and the theory of friction. Next follows Kinetics, or forces not in equilibrium, including the theory of projectiles and falling bodies.

The illustrations are numerous and of a practical kind. Rules are given for the application of the theorems contained in each chapter; and at the end of the work will be found numerous problems of a technical kind, and questions for testing the progress of the student.

In issuing the work we have found it desirable to depart somewhat from our original design, which was to treat the four subjects, Mechanics, Hydrostatics, Hydraulics, and Pneumatics in one volume, uniform with the rest of the series. The extent and importance of the first have, however, induced us to devote an entire volume to it alone; the other three, which are daily increasing in importance, being thrown into a second volume. Then, in order to present at one view the entire subject of which the above four are correllated sections, we propose to publish the two volumes in one, under the title "Natural Philosophy." The complete work, we believe, will be found the most comprehensive, and at the same time compact and practical, text book on the subject yet issued.

J. A. S.

LONDON, *Sept.*, 1873.

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CONTENTS.

NATURAL PHILOSOPHY.

INTRODUCTION.

Definition.—The term Philosophy[1] is derived from two Greek words meaning, a love of knowledge. Natural Philosophy, therefore, means a love of the knowledge of nature. The word Physics,[2] also from the Greek, and meaning nature, is sometimes used in the same sense as Natural Philosophy.

Divisions.—The study of nature in all her manifestations embraces, of course, a very extensive range of subjects, some of which are widely separated from others.

It is found convenient to arrange the various lines of inquiry into separate departments, called *sciences*,[3] the subjects embraced by each department being more or less of a similar character.

The principal divisions of Natural Philosophy are the following :—

I. *Mechanics*,[4] or the science which treats of the laws of motion and force, especially as applied to the construction of machines.

II. *Hydro-mechanics*,[5] or the science of water-machinery, divisible into—

A. *Hydro-statics*,[6] the science treating of the pressure of water.

B. *Hydraulics*,[7] the science treating of the motion of water.

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[1] Gr., *philos*, a lover; *sophia*, knowledge. [2] Gr., *phusis*, nature. [3] L., *scientia*, knowledge. [4] Gr., *mechanē*, a machine. [5] Gr., *hudor*, water. [6] Gr., *istemi*, to stand. [7] Gr., *aulos*, a pipe.

B

III. *Pneumatics*,[1] the science of the pressure and motion of air.

IV. *Acoustics*,[2] the science of sounds.

V. *Optics*,[3] the science of light.

VI. *Calorics*,[4] the science of heat.

VII. *Electricity*,[5] *Magnetism*,[6] and *Galvanism*.[7]

VIII. *Chemistry*,[8] the science of the elements of matter.

IX. *Biology*,[9] the science of life.

X. *Astronomy*,[10] the science of the heavenly bodies.

In this work we are concerned only with Divisions I., II., and III., which are all that are usually included under the term Natural Philosophy, or Physics.

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[1] Gr., *pneuma*, air. [2] Gr., *akouo*, to hear. [3] Gr., *orao*, *optikos*, to see. [4] L., *calor*, heat. [5] Gr., *elektron*, amber. [6] Gr., *Magnesia*, in Asia Minor. [7] It., *Galvani*, the discoverer. [8] Arabic, *kimia*, concealed. [9] Gr., *bios*, life; and *logos*, a discourse. [10] Gr., *astron*, a star; *nemo*, to classify.

PART I.

Mechanics.

Definitions.—In commencing the study of Mechanics it is necessary that the following terms, which are of constant occurrence, should be thoroughly understood.

Matter.—A general term for any kind of substance.

Body.—A portion of matter.

Atom.[1]—A body of the smallest conceivable size, so small that we cannot imagine it to be divided into smaller parts.

Point.—The position of an atom, without regard to its size or shape.

Rest and Motion.[2]—A body constantly occupying the same position, is said to be at *rest;* a body successively occupying different positions, is said to be in *motion.* To illustrate this, let us place a stone on the ground. If left to itself it will constantly remain in the same place; that is, it will continue at rest. If, however, it be not allowed to remain by itself, but some power, such as a push, be brought to bear upon it, it will not continue in the same position, but will move. Motion, therefore, is *change of position.*

Force.—The power employed, as in the preceding example, to move a body, is called *force.*[3]

Hence, whenever we see a body in motion, we may conclude that some force has been used to give it that motion.

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[1] Gr., *atomos*, not to be cut. [2] L., *moveo*, *motus*, to move.
[3] L., *fortis*, strong.

If a force be applied to a body previously at rest, it will cause it to move in a *straight line*, there being no reason why it should incline to one side more than to another. Wherefore, if we see a body moving otherwise than in a straight line, we know that some other than the original force is brought to bear upon it, causing it to deflect from a straight line.

As a body comes to rest sooner when moving along a rough than when moving along a smooth surface, it is evident that its motion is impeded by the resistance it meets with. The less the resistance, the longer will the motion continue; and it may be inferred that if all resistance were removed, the motion would continue for ever.

No terrestrial body, however, will continue to move thus perpetually, but will, after a certain interval of time, return to its previous state of rest, by reason of opposing forces.

Force, therefore, may be defined as *a cause tending to impart motion to a body previously at rest, to change the direction of that motion, or to make it cease.*

Divisions of Mechanics.—All considerations included under the head of Mechanics have relation either to motion or to force, or to the two combined; the subject, therefore, naturally divides itself into two parts, namely:—

I. KINEMATICS, treating of motion; and

II. DYNAMICS, treating of force.

SECTION I.

KINEMATICS.

CHAPTER I.

NATURE OF MOTION.

Definition.—Kinematics is the purely geometrical consideration of motion, apart from the moving body or the force by which the motion has been produced.

In the investigation of motion, means are requisite for ascertaining and describing its character. To this end certain units of measurement, are adopted, showing the relation of motion—(1) to the *space* moved over, (2) to the *mass* of the moving body, (3) to the *time* during which the motion continues.

1. **Units of Space** are of three kinds, having reference to—(*a*) *length*, (*b*) *surface*, and (*c*) *volume*.

(*a*) *Units*[1] *of Length*.—The English standard unit of length is the yard, which is defined to be "that distance between the centres of the two points in the gold studs of the straight brass bar in the House of Commons, at a temperature of 62° F." Any fractional part of this yard may also be taken as a unit, such as a *foot*, an *inch*, or a *line*. To measure length, we have, then, simply to find how many such units are contained in the space to be measured.

(*b*) *Units of Surface*.—The standard unit of surface is a *square* whose side is equal to a unit of length. The number of times this unit of surface, or any of its subdivisions or multiples, as the *square foot*, *square pole*, etc., is contained in a given surface, will represent the extent of that surface.

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[1] L., *unus*, one. Something to call one.

(c) *Units of Volume.*—The standard unit of volume is a cube whose side is equal to a unit of surface. The number of times this unit of volume, or any of its subdivisions or multiples, as the *cubic foot*, *load*, etc., is contained in a given body, will represent the volume of the body.

2. **Units of Mass.**—The standard unit of mass is the *grain*, which is defined to be $\frac{1}{125248}$ of a cubic inch of distilled water at 62° F., 7000 of these grains making a *pound* avoirdupoise. To determine the mass of any body, we must find how many units of mass it contains. This is done by weighing it.

3. **Units of time.**—The standard unit of time is the mean duration of a revolution of the earth upon its axis. This is divided into twenty-four parts, called *hours*, which are again divided into 60ths, called *minutes*, and these again into 60ths, called *seconds*. The second is usually employed in Mechanics as the unit of time.

By means of the above units of measurement, we are able to compare the value of one motion with that of another. Thus, if a railway train, A, has a motion of ten miles an hour, and another train, B, has a motion of twenty miles an hour, we know that the motion of B is to that of A as 2 is to 1; or in other words, that B moves twice as fast as A.

CHAPTER II.

MOTION OF A POINT.

Path.—When a point is in motion, the line described by its successive positions is called its *path*, which may be either straight, curved, or composed of straight and curved lines.

The direction of the motion of a moving point at each instant, is the straight line in which it moves, or the tangent to its position, if its path be a curve.

Velocity.—The rate of motion of a point is called its *velocity*.

Velocity may be represented by lines. Thus, if a line of any length be taken to represent one foot, a velocity of three feet per second will be represented by a line three times as long as the assumed unit. Velocities are of several kinds.

Uniform[1] Velocity.—When a body continues to move over equal units of length in equal units of time, its velocity is said to be *uniform;* and is measured by the number of units of length passed over in a single unit of time. Thus, if a body move at a uniform rate of sixty miles an hour, its velocity may be also expressed as one mile per minute, or eighty-eight feet per second.

Accelerated[2] Velocity.—When a body moves over a greater number of units of length in each succeeding unit of time, the motion is said to be *accelerated*. Again, if the body move over a lesser number of units of length in each succeeding unit of time, its velocity is also said to be accelerated; but it is called *negative accelerated velocity*, or **retardation,** to distinguish it from the former, which is known as *positive accelerated velocity*.

Accelerated velocity may be either *uniform* or *varied*. It is uniform when the increments in successive units of time are equal; and varied when they are unequal.

Varied accelerated velocity "is measured by the increment of velocity which would have been generated in a unit of time, had the acceleration remained throughout that unit the same as at its commencement. The average acceleration during any time is the whole velocity gained during that time divided by the time." —*Thomson.*

Accelerated velocity may be represented as follows:—

Let A be a point at which the body is at rest, and B

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[1] L., *unus*, one; *forma*, kind. [2] L., *accelero*, to hasten.

a point where the velocity of the body equals a certain number of units of length in a unit of time. Then the greater or less acceleration of the velocity may be shown by placing B nearer to or farther from A.

Mean[1] Velocity.—The mean velocity of a body moving with a varied accelerated motion, is that uniform velocity which would carry the moving body the same distance in the same time as the varied motion. This mean velocity may be found by dividing the number of units of length by the number of units of time; and the velocity of such a body at any particular moment is considered to be its mean velocity for a small space commenced at that moment.

Resultant[2] Velocity.—Passengers moving on the deck of a vessel in motion, are influenced by two motions, namely, their motion relative to the vessel, and the motion of the vessel relative to the shore. The true motion of the passengers is the combined effect of these two motions, or what is called their resultant velocity.

When a point has a velocity along a path, and this path has a velocity of its own, the resultant velocity of the point at any position in the path may be represented by the diagonal of the parallelogram constructed at that point by taking, as adjacent sides, the two velocities, each in its own direction (See p. 23). Therefore, when two component motions are in the same straight line, the resultant may be represented by adding or subtracting one motion from the other, according as they are in the same or in opposite directions.

Paracentric[3] Motion.—When a body is moving round a centre, the resolved part of its motion with respect to the centre, that is, the part of its motion by which its distance from that centre is diminished or increased, is called the *paracentric motion*, or spiral motion. When the body increases its distance from the centre at each revolution, its path forms the curve known as an *evolute;*[4]

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[1] Middle, or average. [2] L., *resolvo*, *resultum*, a result.
[3] Gr., *para*, about; *centrum*, a centre. [4] L., *e*, from; *volvo*, to turn.

and when the distance from the centre is decreased, the curve formed by the path is called an *involute*.[1]

Harmonic[2] *Motion*.—"When a point, O (fig. 1), moves uniformly in a circle, the perpendicular, O P, drawn from its position at any instant to a fixed diameter, A A, of the circle, intersects the diameter in a point P, whose position changes by a simple harmonic motion."—*Thomson*.

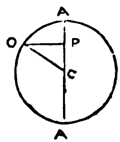

Fig. 1.

Harmonic motion is intimately connected, not only with Mechanics, but with the theories of Acoustics, Light, Heat, etc. Thus the motions of the wires of a pianoforte, when a chord is struck, are harmonic motions; and the motions of the waves of heat, light, sound, etc., are also harmonic. In mechanism we have the rotary motion of a crank harmonizing with the rectilinear motion of the piston-rod.

STRAINS.—An alteration in the relative position of the particles of a body, is called a *strain*. When a cane is bent, the particles on the concave side are forced into closer proximity to each other than when in their normal position; and the particles on the convex side are forced apart. When a heavy weight is suspended from an iron bar, the particles of the rod are torn asunder, as it were, and the rod becomes longer. In the same way, if a heavy weight is supported by a bar, its particles are forced together. All these are examples of strains.

CHAPTER III.

GRAPHIC REPRESENTATION OF MOTION.

Uniform Motion.—By means of diagrams, the motion of a particle may be represented graphically. To illustrate this method, let the following construction be made.

From the point O (fig. 2) draw the horizontal line O X, and the line O Y at right angles to O X. On O X set off a distance, O A, to represent a unit of time, say a second; and on O Y take a length, O P, to represent the

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[1] L., *in*, into.   [2] Gr., *harmozein*, to fit together.

velocity of the moving particle at the beginning of the observation.

At the point A on O X, erect the perpendicular A Q, of such a length as will represent on the same scale the velocity of the particle at the end of the first second. Then on O X take another distance, A B, equal to O A, to represent the second unit of time, and at B raise another perpendicular, B R, representing the velocity at the end of this interval of time; in like manner take another distance, B C, on O X, equal to O A, to represent the third unit of time, and at C erect a third perpendicular, C S, representing the velocity at the end of the third unit of time.

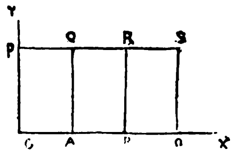

Fig. 2.

Now, if the velocity has remained uniform during these three units of time, the three perpendiculars, A Q, B R, C S, will be of equal lengths, and each equal to O P, representing the initial velocity. Hence the line joining the tops of these perpendiculars is a straight line, P Q R S, parallel to O X. This line, P Q R S, is called the *line of velocities*.

The space which the particle describes after moving for one second is represented by the area of the rectangle O P Q A; in like manner the space passed over in the next second is represented by the equal rectangle A Q R B, and so on. The total space, therefore, described during the three seconds is represented by the sum of the areas of the three rectangles; that is, by the area of the rectangle O P S C.

It is evident that the same construction may be repeated for any length of time, so long as the speed remains unchanged.

Hence it appears that the SPACE *described by a particle moving with uniform velocity may be represented by the area of a rectangle having for one of its sides a line representing* TIME; *and for an adjacent side a line representing* VELO-

CITY; or, in other words, *the number of* UNITS OF AREA *in the rectangle, represents the number of* UNITS OF LENGTH *which the particle passes over in the given time.* The area of a rectangle is the product of one side multiplied by an adjacent side.[1]

The distances on the line O X, as O A, O B, O C, etc., which are *cut off* from the line, are called *abscissæ;*[2] and the perpendiculars A Q, B R, etc., drawn in *order* one after another, parallel to O Y, are called *ordinates.*

Varied Motion.—We will now suppose, that the velocity changes irregularly from time to time. On O X (fig. 3) set off a number of equal spaces 1, 2, 3, 4, 5, 6, each representing a unit of time, say one second; and on O Y set off a number of equal spaces 1, 2, 3, 4, 5, 6, each representing a unit of velocity, say one foot per second. Let a particle move for one second with a

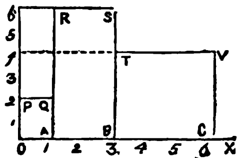

Fig. 3.

velocity of two feet. The space which it passes over may be represented by the area of the rectangle O P Q A. Let the velocity be now increased, and for the next two seconds let the particle move at the rate of six feet per second. A representation of the space traversed in this time is shown in the area of the rectangle A R S B. Now, let the velocity change again, and for the next three seconds let the motion be at the rate of four feet per second. The space during this time will, of course, be represented by the rectangle B T V C. The diagram, therefore, represents the changes of motion under these circumstances during the six seconds; and the space described during the entire time of observation will be

---

[1] Let s = the space, t = the time, and v = the velocity. The relation between these elements, when the motion is uniform, may be expressed by the equation $s = v\,t$.

[2] L., *ab*, from; and *scindo*, *scissum*, to cut.

represented by the sum of the areas of the three rectangles O P Q A, A R S B, and B T V C.

Uniformly Accelerated Motion.—Instead of supposing the motion to be sometimes increased and sometimes decreased, as in the last example, let us now suppose that the motion continues to increase regularly, equal increments of velocity being added in equal successive intervals of time. Such motion is called *uniformly accelerated motion;* and the constant velocity added in each successive unit of time is termed the *acceleration.*

Let a particle start from a position of rest, and move under the influence of a uniform force. Let the abscissa O A (fig. 4) represent one second, and let the ordinate A P represent the velocity added in this one second. At the beginning of the first second, the body is at rest, and therefore has no initial velocity, as at O; but at the end of this second the velocity A P has been imparted to it.

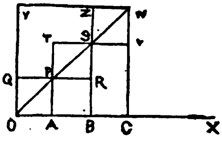

Fig. 4.

Now, if at the end of the first second, the force ceased to act, and the particle continued to move uniformly with the velocity thus acquired, it would during the next second, A B, describe the space represented by the rectangle A P R B. But, as the motion is uniformly accelerated, a velocity R S, equal to A P, is added during the second second. Complete the rectangle P T S R. Make a similar construction for the third second, B C, and join C W; O W must be a straight line. Now, if we suppose that the particle moved during each second with the velocity which it had at the *beginning* of that second, the space described in the first second by the particle starting from rest would of course be nothing; in the next second the space would be represented by the rectangle A P R B, and in the third second by the rectangle B S V C. On the other hand, if we suppose that the particle moved during each second with the velocity which it had at the end of that second, the

spaces described in the first, second, and third seconds, respectively, would be represented by the rectangles O Q P A, A T S B, and B Z W C. But it is evident that neither of these suppositions is quite correct; for the space represented by the *inner* set of rectangles A P R B, B S V C (which is the space described under the first supposition), is manifestly *too small*, while the space represented by the *outer* set of rectangles, O Q P A, A T S B, B Z W C (which is that described under the second supposition), is equally *too large*. The *true* space must therefore lie between the boundaries of these two sets of rectangles. If the intervals of time be taken smaller and smaller, the lengths on O X representing these intervals may be diminished to any extent, and the area of each series of rectangles then approximates more and more closely to the triangular area O W C. Finally, when the intervals of time are taken indefinitely near to each other, the triangle O W C will actually represent the space described by the moving particle. The area of a triangle is calculated by multiplying the number expressing half the base with the number representing the height of the triangle. Hence the space described after three seconds is obtained from the last diagram, by multiplying half O C into W C.[1]

---

[1] Hence, if s = space, v = the velocity generated, and t = the time,

$$s = \frac{v\,t}{2}$$

As an illustration of this, we may refer to the case of a body falling freely under the influence of gravity, which is a uniformly accelerating force.

Such a body will, in one second, fall through about 16·1 feet, and will acquire a velocity of 32·2 feet. That is to say, if the body were removed from the action of gravity at the end of the first second, it would, during each succeeding second, pass through 32·2 feet. But this space is, as we have just seen, exactly double the space actually described in the first second.

It remains to express the space passed over under the action of a uniformly accelerating force in terms of the acceleration and the time. This is easily done, for the acceleration has been previously

Wherefore *the space traversed by a particle moving with a uniformly accelerated velocity, is half that which would be described if the particle moved for the same time with a uniform initial velocity, equal to the velocity attained at the end of the time in the first case.* This is evident, for the area of a triangle is half that of a rectangle having the same base and height.

In the last diagram a uniformly accelerating force was represented as acting upon a particle previously *at rest.* But now, suppose that the particle, before being acted upon by the acceleration, was already in a state of uniform motion. Let this initial velocity be represented by O P (fig. 5). Then, if the particle continued to move uniformly with this motion for three seconds, it would describe the space represented by the rectangle O P Q C. But if during each second it receives the acceleration represented by R S, it will also in three seconds pass over a space represented by the triangle P W Q. Therefore the total space described under the action of both the initial velocity and the uniform acceleration is represented by the area of the figure O P W C, of which the rectangle O P Q C represents the space due to previously

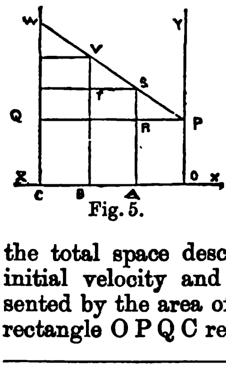

Fig. 5.

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defined to be the velocity added in a unit of time. It is therefore represented in the diagram (fig. 4) by the line A P, or by each of the equal lines R S and V W. The final velocity acquired during the three seconds, is represented by the ordinate C W, which is equal to three times A P. And, generally, the velocity generated by a uniformly accelerated force in t units of time, will be $f\,t$, or $v = f\,t$.

Substituting this value of v in the equation $s = \frac{v\,t}{2}$ we obtain $s = \frac{f\,t.t}{2}$, or, $s = \frac{1}{2} f t^2$. In the diagram, $OC = t$ and $WC = f\,t$; therefore the space s, represented by the triangle O W C, is $\frac{1}{2}$ O C $\times$ W C $= \frac{1}{2}\, t.f\,t = \frac{1}{2} f\,t^2$. This formula, $s = \frac{1}{2} f t^2$ is one of the most important in Kinetics.

acquired velocity; and the remainder, the triangle P W Q, represents the space due to the acceleration.[1]

A four-sided figure with two of its sides parallel, such as O P W C, is called a *trapezoid;* and its area is found by multiplying half the sum of the two parallel sides by the perpendicular distance between them. Here the area of O P W C equals $\frac{1}{2}$ (O P+C W)×O C. The ordinate O P denotes the initial velocity, and C W the final velocity, while the abscissa O C represents the time. Hence, the space described by a particle uniformly accelerated, is obtained by *multiplying half the sum of the initial and final velocities by the time.*

Uniformly Retarded Motion.—The motion of a particle, instead of being uniformly *accelerated,* may be uniformly *retarded.* To illustrate this, let O P (fig. 6) represent the initial velocity. If the moving particle were unacted on by any force, it would in the time O A describe a space equal to the area of the rectangle O P Q A. But the velocity is diminished in this time by a retardation represented by Q R. Hence the space actually described is represented by the area of the figure O P R A; that is, by the area of the rectangle O P Q A *minus* the area of the triangle P Q R; and, using the same symbols as for acceleration, $s=v\,t-\frac{1}{2}\,f\,t^2$. If the velocity had been accelerated, the triangle P Q R would have been added to the rectangle O P Q A, instead of being subtracted as in the present instance. Hence, retardation is called *negative acceleration.*

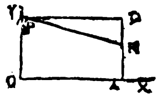

Fig. 6.

Irregularly Accelerated and Retarded Motion.—It remains to show the character of a line of velocities in the most general case; that is, when the motion is *irregularly* accelerated and retarded. In such a case

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[1] It has been proved that the rectangle is represented by $v\,t$, and the triangle by $\frac{1}{2}\,f\,t^2$; therefore the total space is the sum of these quantities; or, $s=v\,t+\frac{1}{2}\,f\,t^2$.

ordinates must be drawn at successive small intervals of time, and the length of these ordinates must of course correspond with the characters of the motion, being greater as the velocity increases, and less as it diminishes. Since the accelerations and retardations occur irregularly, the line of velocities constructed by joining the tops of these ordinates will be a *curve.*

Let the line A B C D E (fig. 7) represent such a curve of velocities.

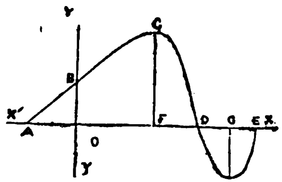

Fig. 7.

An inspection of the diagram shows that at the moment when our observation commences, the velocity is represented by O B; this velocity gradually increases until, at the time represented by the abscissa O F, it has attained a maximum; for of all the ordinates that could be drawn, F C is the greatest. After this instant the speed diminishes, and continues to do so until the instant represented by O D, when the motion becomes retrograde: this is represented by ordinates falling below O X, or becoming *negative.* It is a general convention in such diagrams, that algebraically positive quantities shall be represented by ordinates *above* the axis of abscissæ X X', and negative quantities by those *below;* and that abscissæ to the *right* of the axis of ordinates Y Y' shall be positive, and those to the *left* negative. Hence, the curve shows that after the instant denoted by O D the motion is negative or retrograde, and that this retrogression continues increasing until it attains a maximum at the moment represented by O G. We further see from the diagram, what was the charac-

ter of the motion in *negative time*, that is, before the observation began. The body commenced to move at a time represented by O A prior to our observation, and the motion increased according to the curve A B. In all such cases the area bounded by the curve represents the space described; but the calculation of this area, except in certain cases, is attended with some difficulty.

SECTION II.

DYNAMICS.

CHAPTER I.

INTRODUCTORY.

Definition, etc.—In the previous section we have considered motion from the geometrical point of view, that is to say, motion apart from the body moved. We shall now proceed to consider motion from what may be termed the mechanical point of view, or more exactly, the effects of force upon bodies. The study of these effects constitutes *Dynamics*,[1] which may therefore be defined as that branch of Mechanics which treats of the action of force upon bodies.

The effects of force upon a body result either in rest or motion to the body acted upon. The treatment of Dynamics therefore naturally falls into two divisions.

1. *Statics*,[2] treating of those actions of force upon bodies which bring them to a state of rest, or maintain them in that condition.

2. *Kinetics*,[3] treating of those operations of force which impart motion to the body acted upon. These divisions will be better understood from a consideration of what follows upon equilibrium.

Equilibrium.[4]—If a body at rest be submitted to the action of two or more forces, one of two results must necessarily follow. Either (*a*) the forces are so constituted with regard to their intensities and directions as

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[1] Gr., *dynamis*, force. [2] Gr., *statos*, standing. [3] Gr., *kineo*, to move. [4] L., *æquus*, equal; *libra*, a balance.

to neutralize each other, as in the case of two persons pushing with equal force against opposite sides of a door; or (*b*) they will not neutralize each other, as in the case of two persons pushing on the same side of a door, or on opposite sides with unequal forces.

In the former case (*a*) the body will remain at rest; in the latter (*b*) motion will be imparted to it.

A body under the influence of forces which neutralize each other, is said to be in *equilibrium*. The door in our first supposed case was therefore in equilibrium.

Pressure.—A force acting on a body but not causing it to move by reason of some counteracting force, is called *pressure*.

Superposition[1] of Equilibrium.—It is evident that if to a point already under the influence of forces, two or more forces in equilibrium be added, their addition will have no more effect than if applied to a point upon which no forces were acting. This addition of forces in equilibrium is called the *superposition of equilibrium*.

From these considerations it is apparent that two sets of problems arise concerning the effects of forces, viz.:—

1. *To ascertain that relation between the directions and intensities of forces acting on a body which will keep the body in equilibrium.*

The solution of these problems is called Statics.

2. *The directions and intensities of forces not in equilibrium being given, to determine the direction and velocity of the motion thereby imparted.*

The solution of these problems is called Kinetics.

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[1] L., *super*, above; *pono*, *positum*, placed.

DIVISION I.

Statics.

CHAPTER I.

STATICAL FORCE.

Definition.—Statics is that branch of dynamics which treats of bodies in equilibrium, and statical force is that operation of force which produces equilibrium.

Direction and Intensity of Forces.—In estimating the effects of a force, two things must be taken into consideration; first, the *direction* of the force, second, its *intensity*.

The direction of a force is the line in which it tends to produce motion, or the line along which a body acted on by the force would move if free to do so.

The intensity, or strength, of a force is expressed by adopting some unit as a standard, and stating how many times the unit is contained in the given force.

The unit of force is generally taken at a pound weight. Hence a force of six pounds is said to be twice as great as a force of three pounds. Statical forces are therefore measured by the pressure they exert.

Equal Forces.—Forces are said to be equal if their intensities are such that when applied to a free point in opposite directions, they produce equilibrium.

Two equal forces applied to a point in the same direction, are equal to a single force which is double of either.

Two unequal forces applied to a point in the same direction are equal to a single force whose intensity is equal to the *sum* of the intensities of the two forces.

Wherefore, two unequal forces applied to a point in opposite directions are equal to a single force whose intensity is equal to the *difference* of the intensities of the two forces, and whose direction is the same as that of the larger.

Representation of Force.—It is often convenient, in treating of forces, to represent them to the eye, or give a *graphic*[1] representation of them, as it is called. By this means even complicated arrangements are readily grasped, which would be very perplexing if merely described in words. This representation is most easily obtained by means of lines. Thus, if we take a line of one inch to represent a force with an intensity of one pound, a line of three inches will represent a force having an intensity of three pounds; and, moreover, the direction of the line will represent the direction of the force, and an arrow-head will show which way it acts.

Transmissibility[2] of Force. Let A C (fig. 8) represent a bar of iron, and let a force of two pounds be applied at C, so as to act upon A. Then the force of C on A is two pounds, or the same as if applied directly to A. And further, if the bar be lengthened to B, and the force be applied at B, the effect will still be the same.

Fig. 8.

This is only true, however, on the supposition that the bar of iron remains unaltered. For it is clear that if part of the force be expended in altering the condition of the particles composing the bar, the force acting on A will be diminished,

This power of transferring a force from one point to another is called the transmissibility of force, and may be thus described :—

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[1] Gr., *grapho*, to write. [2] L., *trans*, through; *mitto*, to send.

A force may be transferred from its original point of application to any other point in the same direction, without change of effect, provided the second point be rigidly[1] *connected with the first.*

Resultant of Forces.—If two or more forces act upon a point so as to cause it to move, their combined effect is equal to that of a single force acting in the direction in which the point is moved.

This single force may be substituted for the original forces, and will produce the same effect. It is called the *resultant*,[2] and the original forces are called the *components.*[3]

The resultant may therefore be described as the single force which, if applied to the same point as its components, will produce the same effect.

Hence, if to a number or system of forces a new force be added, equal to their resultant and opposite in its direction, this new force will keep in equilibrium the point upon which the system of forces is acting.

If two forces act upon a point in the same straight line and *in the same direction*, their resultant is a force equal to the sum of the two forces in their direction.

Hence, if two forces act upon a point in the same straight line *in opposite directions*, their resultant is a force equal to the difference between the two forces, in the direction of the greater force.

Composition and Resolution of Forces.—The process for finding a single force which shall be equal in its effect to two or more forces, is called the *composition of forces.* The inverse process, namely, that of finding two or more forces whose effect shall be equal to that of a single force, is called the *resolution*[4] *of forces.* The former process is the finding of the resultant when the components are given, and the latter the determining of components when the resultant is given.

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[1] L., *rigidus*, stiff. [2] L., *re*, back; *saltandum*, leaping. [3] L., *compono*, to place together. [4] L., *re*, again; *solvo*, to melt.

Parallelogram of Forces.—If two forces acting on a point be represented by straight lines, as above described, and a parallelogram be constructed having these lines as adjacent sides, the resultant of the two forces will be represented in direction and intensity by the diagonal of the parallelogram drawn from the point. The parallelogram so constructed is called the *parallelogram of forces.*

In determining the truth of the above proposition, we shall first consider the case of equal forces, and secondly, that of unequal forces.

1. *Equal Forces.* Let A B, A C (fig. 9) represent two equal forces acting upon A. Now, since A B and A C are equal, half their effect to move A in their respective directions is neutralised. That is to say, the force A B prevents the force A C from moving A in the direction A C; and in the same degree A C prevents A B from moving A in the direction AB. Therefore the resultant must be in the direction of the line bisecting the angle B A C. And as the sides of the parallelogram are equal, the diagonal A D bisects the angle B A C.

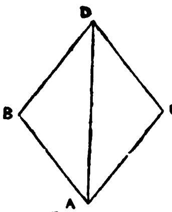

Fig. 9.

The diagonal is, therefore, in the *direction* of the resultant.

Let us now show that the diagonal A D represents the resultant in *intensity.*

Let A B C D (fig. 10) again represent the parallelogram of the forces A B, A C, acting upon A. From A draw A F equal to the supposed resultant A D. Complete the parallelogram A F E B, and join A E.

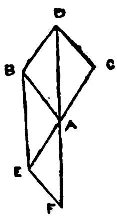

Fig. 10.

As A F is equal to the resultant of A B and A C, and acts opposite to them, the system of forces A B, A C, and A F about A is in equilibrium. One of the forces, A C, also

balances the others, A B and A F; it must therefore be equal and opposite to their resultant. This resultant is in the direction A E, because B A F E is a parallelogram. Therefore A C and A E must be in the same straight line. Therefore A E B D is a parallelogram, and A D is equal to E B. But A F is equal to B E. Therefore A F is equal to A D; and A F being assumed to be equal to the resultant, A D is that resultant.

The diagonal A D, therefore, represents in *intensity* the resultant of the two forces A B, A C.

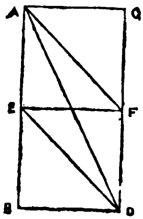

Fig. 11.

2. *Unequal Forces—First.* Let one force, A B (fig. 11), be *double* the other, A C. Complete the parallelogram A D, bisect A B in E, and draw E F parallel to A C.

Join A F and E D.

The force A B is equal to the two equal forces A E, E B. The resultant of A C, A E, is in the direction A F, and the resultant of E F, E B, is in the direction E D. By resolving the force A F at the point F it is equivalent to the two forces C F, E F. The force C F passes through the point D in the resultant of the two forces E F, E B. If therefore the point D remains immovable, the forces C F, E F, and E B are destroyed. But C F and E B are equal to the force A B, and E F is equal to the force A C. Therefore the point D is in the resultant of the forces A C and A B. The point A is another point in this resultant. Wherefore a line joining A and D is in the direction of the resultant of the two forces A C and A B.

Second. Let one force be a *multiple* of the other, and let the force A B (fig. 12) contain four units, and the force A C three units. Complete the parallelogram A B D C, and from the points *a*, *b*, and *c*, draw *a a′*, *b b′*, *c c′* parallel to A C, and from *d* and *e* draw *d d′* and *e e′* parallel to A B. Because the forces A *a* and A *d* are equal, their resultant lies in the direction A E;

and because A d and $d\,e$ are equal, the resultant of A a and A e is in the direction A F. And since e F is equal to e C their resultant is in the direction $e\,a'$; therefore the resultant of A a and A C is in the direction A a'.

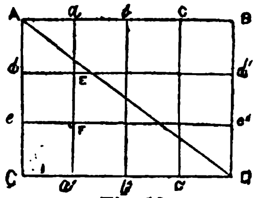

Fig. 12.

In like manner it can be proved that the resultant of $a\,b$ and $a\,a'$ is in the direction $a\,b'$; therefore the resultant of A b and A C is in the direction A b', and so on until the resultant of A B and A C is proved to be in the direction A D.

Third. Let the forces A B, A C (fig. 13) be *unequal* and also *incommensurable.* Their resultant lies in the direction A D. If it be possible, let A G, intermediate between A C and A D, be the direction of the resultant. Let a measure of A C be taken, less than D G, and measured into A B as often as possible to the point P, leaving a remainder, P B, smaller than D G. Draw P R parallel to B D. Since the forces A P and A C are commensurable, their resultant lies in the direction A R; which resultant, compounded with the remainder P B, will give a final resultant of the forces A B and A C lying in the angle B A R, and therefore, *a fortiori* intersecting the line C D in the point D outside the point G. Consequently A G is not the resultant of the forces A B and A C; and it can be shown that the resultant can have no other direction than A D, which therefore is the direction of the resultant of A B and A C.

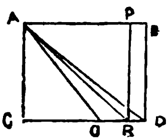

Fig. 13.

Method of Resolving Forces.—Having proved that the resultant of two forces not in the same straight line but acting upon a given point is in the direction of the diagonal of the parallelogram, and that its intensity

is represented by the length of the diagonal, we are in a position to consider the simpler problems of the decomposition and resolution of forces.

To resolve a force, A B (fig. 14), into two forces acting upon the same point as the given force and in given directions.

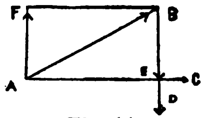

Fig. 14.

Let A B (fig. 14) be the given force. From A draw A C in the direction of one of the component forces, and from B draw B D in the direction opposite to that of the other component force. B D and A C intersect in E. Complete the parallelogram A E B F. The forces A F, A E, have as their resultant A B; and they act in the given directions, and are therefore the required components.

The resultant can always be determined by Trigonometry; but as it is our wish to avoid the use of that science in this work, we shall confine our examples to those cases that can be solved by easier methods.

1. *When the angle of the forces is a right angle.* Let A B (fig. 14*a*) be 6 lbs., A C, 8 lbs., and let the forces act at

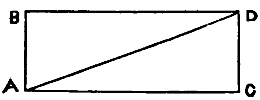

Fig. 14*a*.

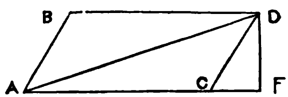

Fig. 14*b*.

right angles. Complete the parallelogram, and draw the diagonal A D, which will represent the resultant. Now $AD^2 = AB^2 + AC^2$. Therefore, $AD^2 = 6^2 + 8^2 = 100$. Consequently $AD = \sqrt{100} = 10$.

2. *When the angle of the forces is* 60°. Let A B (fig. 14*b*) = 4 lbs., A C = 3 lbs., and the angle B A C = 60°. Complete the parallelogram, and join A D. From D draw D F, at right angles to A C produced to F. Because B A C = 60°, D C F = half an equilateral triangle. Therefore C F = half

$CD=2$. Wherefore $DF^2=4^2-2^2=12$. Now $AD^2=DF^2+AF^2=12+25=37$. Therefore $AD=6{\cdot}08$.

3. *When the angle of the forces is* 30°. Let A B (fig. 14*c*)

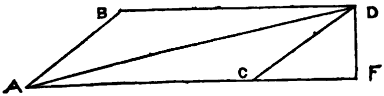

Fig. 14*c*.

=4 lbs., A C 3 lbs., and the angle $BAC=30°$. By the same construction as in the last case, $DF=2$, $CF=\sqrt{12}=3{\cdot}46$. Then $AD^2=DF^2+AF^2=4+6{\cdot}46^2=46$ *nearly*. Therefore A D equals 6·78 nearly.

Polygon of Forces.—We will now suppose the point A to be acted upon by several forces, A B, A C, A D, and A E (fig. 15). On A B take any distance, A F; through F draw F G parallel to A C, and of such a length that F G : A F : : force A C : force A B. Again, through G draw G H parallel to A D, and so proportioned that G H : G F as force A D : force A C. So also draw H K parallel to A E, so that H K : H G as force A E : force A D, and join K A. A single force acting in the direction K A or A L, and having the same ratio to each of the other forces as A K has to that side of the polygon which is parallel to that force, will produce an effect on A equal and opposite to the combined effects of the forces B C D E. This may be proved by finding the resultant of the first two forces, and then the resultant of the next force and that resultant, and so on, whatever be the number of forces.

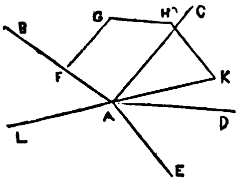

Fig. 15.

Wherefore:—*If a number of forces acting on a point are parallel and proportional to all the sides of a polygon taken in order, except one; a single force, in the same*

proportion and direction as the remaining side, will be the resultant of all the other forces.

Also:—*Whenever any number of forces, acting on a point, are so disposed that they can be represented in magnitude and direction by the sides of a polygon, or other geometrical figure, taken in order, the forces are in equilibrium.*

Hence it follows that:—*As one side of any geometrical figure is less than the sum of the remaining sides; so a mechanical effect can be more economically produced by a single force in one direction, than by a number of forces acting in various directions.*

Parallelopiped of Forces.—The above problems refer to forces acting in the same plane. If, however, the forces act in the direction of the sides of a parallelopiped, the resultant is in the direction of the diagonal of the parallelopiped.

This is a necessary corollary to the parallelogram of forces, for the resultant of two of the forces is the diagonal of that face of the parallelopiped of which they form the sides; and the resultant of the third force, with the resultant force of the first and second forces is the diagonal of the parallelopiped through the point of application.

Parallel Forces.—We will now proceed to the examination of forces acting upon more than one point of a rigid body, and will commence with such as act parallel to each other, and are therefore called *Parallel Forces.*

Two sets of problems arise in connection with parallel forces; in the first, the forces act in the same direction; in the second, they act in opposite directions.

1. *Forces Acting in the Same Direction.*—Let A and B (fig. 16) be the forces acting in the same direction upon the points C and D of the rigid body C D.

Introduce two equal forces, E and F, acting in opposite directions, into the system, and compound them with the forces A and B. Their lines of resultants

will intersect in K. From K draw K L M, parallel

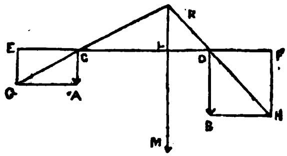

Fig. 16.

to C A or D B, cutting C D in L. The forces E and A may be transferred to K in a direction parallel to themselves, so also may F and B. Since the forces E and F are equal and opposite, they have no effect, and the forces A and B act along K L M. The forces A and B have therefore a resultant, L M, parallel to themselves and equal to their sum.

This may be also proved by suspending two weights, A and B (fig. 17), from the extremities of a rigid bar,

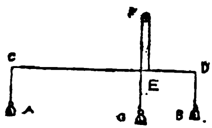

Fig. 17.

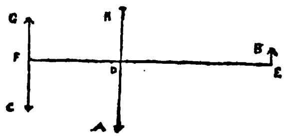

Fig. 18.

C D, and attaching a weight, G, equal to their sum, by a cord passing over a pulley, F, and fastening the cord to the point E, so that C E : E D as the weight B : the weight A. The system will then be found to remain in equilibrium.

To find the length of an arm of a system of concurrent parallel forces: *as the sum of the forces is to one of the forces, so is the distance between them to the arm of the other force.*

2. *Parallel Forces Acting in Opposite Directions.* Let A and B (fig. 18) be the forces, and let A be the greater. Find a force, C, equal to the difference between A and B.

Produce the line E D to F, so that E D : D F : : C : B. At the point F apply the force C in the direction, D A, of the greater force, A. The force, C is the required resultant.

Take F G equal and opposite to F C. The resultant of E B and F G is a force D H, equal to the sum of E B and F G; but by construction D A is equal to F G and E B; therefore, D A is equal to D H. And since D A is opposite and in the same straight line, it balances D H. Wherefore F G equilibrates E B and D A, and therefore F C, its equal, is the resultant of E B and D A.

From a consideration of these cases, the following conclusions may be drawn:—

1. *Parallel forces, acting in the* **same** *direction, applied to two points of a body, have a resultant equal to their sum.*

2. *Parallel forces, acting in* **opposite** *directions, applied to two points of a body, have a resultant equal to their difference.*

3. *When the parallel forces are equal, the position of the resultant is in the centre of the line joining the points of application of the forces.*

4. *When the parallel forces are unequal, the position of the resultant is a point in the line joining the two points of application of the forces, having a position which varies in the inverse ratio of the forces.*

5. *In any system of parallel forces, a point of application of a resultant may be found. This point will be the centre of the system.*

Moments.—The product of a force by the perpendicular upon its line of action from any point, is called the *moment*[1] of the force with respect to that point.

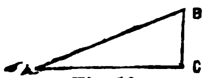

Fig. 19.

Thus—

Let A (fig. 19) be the point, and B C the force. The moment of the force about A is $BC \times AC$. If A C be con-

[1] L., *moveo, momentum*, to move.

sidered to be the base of a triangle of which B C is the perpendicular, the moment of the force B C about A is represented by twice the area of the triangle B A C.

It is evident that the moment of a force about a point in its own line of direction is zero.

When two forces act on a point, their moments, with respect to any point in their resultant, are equal.

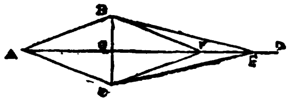

Fig. 20.

Let A B, A C (fig. 20), be forces acting upon the point A, and let A D be their resultant. Take any point, E, in the resultant, and join E B, E C. We shall prove the moments about E to be equal, if we show that the triangles A B E, A C E are equal.

Complete the parallelogram A B F C, and from the points B and C draw perpendiculars B G, C H. We know that perpendiculars drawn from the angles of a parallelogram to the diagonal are equal; wherefore, B G = C H, and the base, A E, is common to the two triangles A B E, A C E; wherefore, the triangles are equal, and the moments of the forces A B and A C about A are equal.

The converse of this theorem is equally true, viz. :—

When the moments of two forces meeting in a point are equal with respect to any point lying in their plane, that point must be in the resultant.

The effect of a moment of force about any point, therefore, tends to turn or twist the point of application of the force about the point.

Hence, when two forces meeting in a point are applied to a body, every point of that body, except those situate on the resultant, is subjected to a twisting force equal to the sum of the twisting moments of the two forces.

A line may be drawn, along which the twisting actions are opposite and equal, which line is the resulttant of the forces. "The moment of a force is, therefore, the numerical measure of its importance."—*Thomson.*

Couples.—When two equal, parallel, and opposite forces are applied to two points in a body, their effect is to twist the body, and not to move it along any resultant path. Such a combination of forces is called a *couple.*[1] The perpendicular distance between the lines of action of the two forces is called *the arm of the couple;* the perpendicular to the plane of the couple at the middle point of the arm, is called *the axis of the couple.*

The moment of a couple is the product of either of its forces by its distance from the axis.

A railway turn-table is an illustration of a couple. If equal forces be applied at either end in opposite directions, their effect is to turn the table round its centre, and not to move it in a straight line.

If equal forces be applied at a certain distance from the centre, and then the same forces be applied at double the distance, their moments are doubled. Also, if the distances remain the same and the forces are doubled, the moments are also doubled. The effect of a couple is measured by the moments of the forces about the axis; and while the moments remain the same, no change in the couple will alter its effect. Wherefore—

A couple may be turned in its own plane through any angle without altering its effect.

A couple is not altered by being moved parallel to itself.

Two couples are equivalent, if their moments are equal and act in the same direction.

Since a couple cannot have a single force as a resultant, no single force can counteract the effect of a couple;

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[1] L., *copula*, a link.

but a system of couples may have a resultant couple, which may be found by the same methods as the resultant of forces.

Action and Reaction.—When a man in a boat pushes with his boat-hook against a quay, the boat is driven away from the quay with a force equal to that with which the man pushes against the quay. Similarly, all bodies acted upon exert a force called *reaction*, which is equal and opposite to the original force. In other words, *action and reaction are equal.*

It follows, therefore, that as every force is accompanied by an equivalent reaction, every machine exerts a reacting force equal and opposite to the force exerted upon it; the force represented by the power being exactly balanced by the reaction due to the weight, as will be abundantly shown in treating of the mechanical powers. And since friction, the weight of the machine, and other resistances have to be overcome in addition to the weight to be moved, the employment of machines, instead of diminishing, has really the effect of augmenting the force necessary to overcome a given resistance. Why, then, are machines used at all? Because it is more convenient to exert a moderate force over a proportionately longer time, than to exert a greater force over a proportionately shorter time. Thus, it is more convenient to carry forty half-hundred-weights a certain distance, occupying forty minutes of time, than to carry a ton the same distance in a single minute.

CHAPTER II.

GRAVITY.

Definition and Divisions.—Every particle of matter has a tendency to draw to itself every other particle, and this tendency is called the *force of gravity*.[1]

[1] L., *gravis*, heavy.

The force of gravity may be conveniently studied under three heads; namely:—

1. *Universal*[1] *Gravity*, which is the force exerted by every body upon every other body;

2. *Terrestrial*[2] *Gravity*, or the gravity of the mass of the earth, as compared with that of bodies upon it; and—

3. *Relative, or Specific*[3] *Gravity*, or the force exerted upon different kinds of matter by the earth. As this kind of gravity is intimately connected with the subject of Hydro-mechanics, we shall postpone the consideration of it until we treat of that subject.

1. **Universal Gravity.**—Universal gravity is that force which causes all particles of matter to attract one another. It therefore includes the other two kinds. Its effects are to be seen everywhere. If we place two light bodies, such as leaves, near each other in a basin of water, this force will manifest itself by the movement of the leaves towards each other. Two ships, becalmed, will gravitate, as it is called, towards each other, although miles apart in the first instance. It is the force of universal gravity which keeps the planets from flying off into space. The force in this case is called *centripetal*,[4] because it tends to draw the body towards the centre round which it moves; as opposed to *centrifugal*[5] force, which is shown in the tendency of a revolving body to fly from the centre. The equilibrium of these two forces keeps the planets in their orbits,[6] and likewise our sun with other similar suns in their own particular paths round the centre of the universe.

As all matter possesses this force of gravity, it follows that the sun gravitates towards the earth, and the earth towards the moon. When the moon in her orbit is

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[1] L., *universus*, the whole. [2] L., *terrestris*, belonging to the earth. [3] L., *species*, likeness; relating to bodies of like nature. [4] L., *centrum*, the centre; and *peto*, to seek. [5] L., *centrum;* and *fugo*, to fly. L., *orbita*, a wheel-track.

progressing towards the sun, her motion is accelerated; and when from the sun, her motion is retarded, the gravity of the sun becoming in the one case an accelerating, and in the other a retarding force.

The planets exert a gravitating force upon each other, hence their orbits are not true geometric figures. When we see a free body moving round a centre, in a path differing from a circle, we conclude that the irregularity is due to the presence of a gravitating force exercised by another body. The motion of the planet Uranus was observed by Adams and Leverrier to be irregular, and they ascribed it to the gravitating effect of an undiscovered planet. After calculating and announcing the position this planet ought to occupy to produce such an erratic motion, it was at once discovered, and is called Neptune.

2. **Particular, or Terrestrial, Gravity.**—Particular, or terrestrial, gravity is that section of universal gravity exerted by the mass of the earth; or, in other words, that force which tends to draw all bodies towards the earth's centre.

The reason why the earth possesses this property is evident when we remember that all the particles of which the earth is composed possess their share of gravity. The result of this is, that the force exerted by the whole earth is the same as if the gravity of every particle were concentrated at or near the centre of the globe, which is therefore called the *centre of gravity of the earth.* This point is, in fact, that in which the resultants of the force of all the particles meet.

As the intensity of the force of gravity of a body is proportional to its mass, it follows that the effect of the gravity of the earth upon any terrestrial body is vastly in excess of that of any terrestial body upon it. This force of the earth is at once manifest when a body is unsupported, it being at once drawn to the earth, or it *falls*, as we term it. The intensity of the force with which the earth attracts a body is called the *weight* of the body.

The moon is prevented from flying off into space by the equilibrium existing between her centrifugal force and the force of terrestrial gravity.

As every body possesses a force of gravity of its own, it may be asked: Why does not the earth move towards the falling body, as it must necessarily be pulled by it? The answer is, that the earth *does* move towards the falling body; but that the motion imparted to the earth by the falling body is as much less than the motion imparted to the falling body by the earth as the mass of the falling body is less than the mass of the earth. Now, the mass of the earth is 5852 billions of tons; and as the falling body can be at the most but a few tons, and probably does not exceed a few pounds, its effect upon the earth is practically nothing.

Parallelism of Gravity.—As the force of gravity acts in straight lines towards the centre of the earth, these lines are necessarily converging lines, the point of convergence being the centre of gravity of the earth. But as the size of the earth is very great, and as we can only observe the effects of gravity upon bodies through a small space, extending from a limited distance above to a limited distance beneath the surface, and covering but a limited area, the amount of convergence is imperceptible. We may therefore consider the force of gravity as acting upon any particular body in parallel lines, without involving any appreciable error.

Centre of Gravity.—It has been shown[1] that in every system of parallel forces there is a central point, to which, if the resultant be applied in an opposite direction, equilibrium will be produced. This point, with regard to the gravity of a body, is called the *centre of gravity* of the body, and will be hereafter represented by the letters C G.

Method of finding the centre of gravity.—The C G of any body may be thus found.

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[1] Page 29.

Suspend the body by any point, A (fig. 21); a vertical line from this point will pass through the body in the direction A B, and the centre of gravity of the mass must be somewhere in this line.

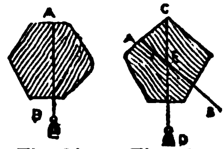

Fig. 21. Fig. 22.

Now suspend the body by another point, C (fig. 22). The vertical from this point will pass through the body in the direction C D; and the point, E, where the two lines intersect is the centre of gravity of the body. A force acting on this point has the same effect as if applied to all the particles of the body collectively.

The centres of gravity of sundry important figures are here given:—

1. *Of a triangle.* That point of intersection of two middle lines, or that point in the line joining the middle of the base with the opposite angle, which is one-third of its length from the base.
2. *Of a semicircle.* At a distance from the base found by dividing two-thirds of the square of the diameter by the circumference.
3. *Of a semi-ellipse.* Same as a semicircle of the same height.
4. *Of a parabola.* Three-fifths of the height.
5. *Of a cycloid.* Seven-twelfths of the height.
6. *Of a sector of a circle.* At a distance from the centre found by multiplying two-thirds of the radius by the chord, and dividing by the arc.
7. *Of a quadrant.* At the same distance from either radius as that of the semicircle is from its base.
8. *Of the surface of a hemisphere.*[1] At the middle point of the height.
9. *Of a prism or cylinder.* The middle point of the line joining the centres of gravity of the two ends.
10. *Of a pyramid or cone.* That point on the line joining the centre of gravity of the base with the apex, which is one-fourth of its length from the base.
11. *Of a hemisphere.* At three-eighths of the radius.

To find the C G of two particles, A and B: Join A B. Bisect the line A B in C, so that the weight of A is to the

[1] Gr., *hemi*, half; and *sphaira*, a ball.

weight of B as B C is to A C. The point C is the C G. In a similar manner the C G of any number of particles may be found.

The truth of the following axioms will now be apparent.

1. *The direction of the force of gravity is always vertical; that is, it is perpendicular to the plane of the horizon at any place.*

2. *In all places equidistant from the centre of gravity of the earth, the force of gravity is equal.*

3. *As the force of gravity increases with proximity to the centre of the earth, the motion of a falling body is an accelerated motion.*

Stable and Unstable Equilibrium.—A body, when suspended by a thread from a single point, so arranges itself as to bring the C G immediately beneath the point of suspension. It is, therefore, in equilibrium; for the force of gravity and the opposing force of the cord are equal and opposite forces, acting upon the same point, namely the C G.

Also, when a body at rest is supported by one point, the C G lies in the vertical above that point.

When a body at rest is supported by more than one point, the resistances of the points constitute parallel forces, and will, therefore, have a resultant parallel to themselves. Therefore, if the points be joined by lines, the resultant will fall within the lines. Hence we arrive at the following important axiom:—

The vertical through the C G of a body at rest falls within the base.

When a body is at rest, the force required to move it varies with the position of its C G. Let A B C D (fig. 23) be a block whose C G is at E. If the block be turned on one edge, D, the C G will ascend the arc E E′. It is clear that the force required to turn the block decreases as its C G rises. When its C G arrives at E′ in the vertical

Fig. 23.

through the point of support, D, the body is in equilibrium, or at rest, D then being the base over which the C G is placed. In this position, however, the equilibrium is very different from that which obtained when C D constituted the base. For, whereas, in that case, considerable force was necessary to move the body out of its state of equilibrium, the slightest force is sufficient to overbalance it now that the C G is at E′; because any movement whatever will throw the C G on one side or other of the base, in which case the block will be no longer in equilibrium, and will consequently fall.

A body supported as the block A B C D upon C D, is said to be in *stable equilibrium.*

A body supported as A B C D upon D, is said to be in *unstable equilibrium.*

In stable equilibrium the C G occupies the lowest possible position; and in unstable equilibrium, the highest.

We may therefore define a body in stable equilibrium, as one whose C G occupies the lowest possible position; and a body in unstable equilibrium, as one whose C G occupies the highest possible position compatible with equilibrium.

Neutral Equilibrium.—In a globe or cylinder having the same density throughout, and resting upon a horizontal plane, the C G is always at the same height, in any position of the body. Under these circumstances the body is said to be in a state of *neutral equilibrium.*

Hence it follows that of bodies of equal mass, those are easiest to move which are globular or cylindrical in shape.

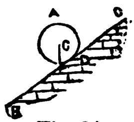

Fig. 24.

Inclined Planes.—In the case of a globe or cylinder resting upon an inclined plane, as A upon B C (fig. 24), the vertical from the C G, marked G in this and the following figures, falling on one side of the point of support, or base, D, the globe will not be in equilibrium.

As in every position the vertical from the C G falls outside the base, the globe will continue to roll down the inclined plane. It is therefore impossible for a globe or cylinder lying on its side to be in equilibrium on an inclined plane.

In the case of a prism upon an inclined plane, the state of equilibrium or otherwise of the body is determined by the height of its C G. Let D (fig. 25) be a

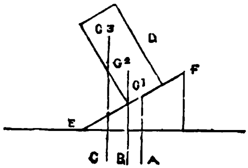

Fig. 25.

prism resting upon the inclined plane E F, and G^1 its C G. Because the vertical G^1 A falls within the base, the prism is in stable equilibrium.

Now let G^2 be the C G. The vertical G^2 B passes through the angle at the base of the prism, and is therefore in the highest position in which the prism can remain in equilibrium. The prism is therefore in unstable equilibrium.

Lastly, let G^3 be the C G. Because the vertical G^3 C falls outside the base, the prism is no longer in equilibrium, and will overturn.

However unstable a body may appear to be, provided the vertical from the C G falls within the base, the body will stand. The leaning tower of Pisa may be given as an example. Although leaning several degrees from the perpendicular, yet, as its C G falls within its base, it is quite stable. Passengers at sea are at first unable to walk with ease because they cannot bring their C G over their feet, which constitute their bases. A man feeling himself to be falling, unconsciously puts out his foot, thereby enlarging his base, and so bringing his C G above it. A man walking along an inclined path must

bring his C G to the vertical, or he will fall. Hence, sailors walking along a ship's deck that is much inclined, appear to be leaning towards the higher side. Wagons, heavily loaded on the top are more liable to be overturned than when the weight is kept low down.

CHAPTER III.

LAWS OF MOTION.

Before entering upon the subject of machines, certain laws appertaining to motion ought to be understood. These are called the laws of motion, and will now be readily comprehended, as their principles have already been explained.

First Law of Motion.—*A body will continue in a state of rest, or of uniform motion in a straight line, unless compelled to alter its state by force impressed upon it.*

The proof of this law is exceedingly simple; for it is evident that a body at rest can have no tendency to move of itself. And, as a moving body continues in motion longest upon a smooth surface, we may infer that, if the resistance offered by the roughness of the surface were entirely removed, together with the force of gravity and the resistance of the air, the body would move for ever. Furthermore, there being no inherent tendency in the body to move to one side more than the other, its path would be a straight line.

Inertia.—This property of a body maintaining its condition of rest or motion if not acted upon by force, is called *inertia*;[1] and the First Law of Motion is often spoken of as the *Law of Inertia.*

Second Law of Motion.—*When a force acts upon a body in motion, the change of motion is the same in magnitude and direction as if the force acted on the body at rest.*

If a seaman at the masthead of a steamer moving at

[1] L., *iners*, idle.

a certain speed, drop a knife, it will fall at the foot of the mast, the same as if the vessel were at rest. It is evident, therefore, that the knife partakes of the motion of the vessel whether it is held in the man's hand or is falling through the air. The time of its descent will be the same as if the vessel had no motion. In other words, the horizontal motion of the vessel and the vertical force of gravity exercise their influence upon the knife irrespective of each other.

As another illustration, let the force with which a ball must be thrown to reach the top of a railway carriage at rest be known. When the carriage is in motion, the same force only will be required to propel it to the roof, although the relative motion of the ball with regard to a fixed spot will be different.

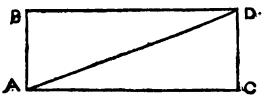

Fig. 26.

Thus, in fig. 26, let A B be the distance between the hand and the roof of a railway carriage. The force necessary to project a ball from A to B is the same whether the carriage be at rest or in motion; for the distance A B remains precisely the same. But suppose that during the time occupied by the ball in going from A to B the carriage has moved from A to C; the path described by the ball, with respect to a fixed point, will be in the direction of the diagonal A D, but it will pass from A to D in the same time as from A to B.

These two cases illustrate the truth of the second law of motion, for the change of motion in the cases of the knife and ball were the same in magnitude and direction as if the forces acted on the bodies at rest.

Upon these two laws of motion the theory of the movements of the heavenly bodies is founded. All calculations respecting moving bodies deduced from these laws are verified by the observed motions

of the planets and by other astronomical phenomena. This affords the strongest proof of the accuracy and truth of the laws.

Third Law of Motion.—*To action there is always an equal and opposite reaction.*

The subject of this law has been treated in Chap. II. of Statics; we shall not, therefore, discuss it at length in this place.

When two equal bodies having the same velocity and moving in opposite directions, come into collision, they destroy each other's motion. But if a body in motion come into collision with a body at rest, the moving body will rebound after the contact, thereby showing that the body at rest returned the blow given by the moving body. It will be shown, in treating of Impact,[1] that this return blow is equal to the first.

CHAPTER IV.

MACHINES.

Mechanical Agents.—We have now to consider those practical applications of force which are made to serve the purpose of mankind. It being impossible to create force, we can only apply such forces as already exist in nature. These are chiefly the strength of men or animals, the force of air or water in motion, steam, the weight of water, electricity, magnetism, and so forth. To all such forces as are applied to man's use, the name of *mechanical agents* is given.

Definition of Machines.—It seldom happens that the force we have at command acts in the way we wish to use it. Some contrivance therefore has to be found to adapt the force to meet our requirements. All such contrivances are called *machines.*[2]

A machine may be defined as an instrument for transmitting the power of a mechanical agent from one point

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[1] Page 141. [2] L., *machina*, a contrivance.

to another, or, in other words, for changing the direction, velocity, or intensity, of a motion which is the result of a mechanical agent.

Varieties of Machines.—Although the variety of machines is almost infinite, and their uses equally diversified, yet it will be found that every machine, whether complex or simple, can only be designed to produce one or more of the following effects:—

1. *To change the* DIRECTION *of the force employed.*
2. *To change the* VELOCITY *of the force.*
3. *To change the* INTENSITY *of the force.*

Power and Weight.—In speaking of machines, we shall consider the force employed and the resistance to be overcome to be represented by equivalent weights; the weight representing the moving force being called the *power*, and the weight representing the resistance being called the *weight.*

Unit of Force.—Whatever kind of force we employ, its power is measured by the number of units of force it contains. The unit is generally taken at one pound; so that when we talk of a horse pulling with a force of 150 lbs., or the wind blowing with a force of 5 lbs. to the square foot, we mean that the horse pulling at one end of a rope passing over a pulley would just balance a weight of 150 lbs. at the other end, and a pressure of 5 lbs. applied to a square foot of surface would exactly counteract the force of the wind on the opposite side of the square foot of surface.

Omitted Resistances.—In investigating the transmission of force from the power to the weight through the medium of machines, it will be convenient to omit several resistances which oppose the free transmission of the force, and the calculation of which would become extremely complex, and present difficulties which would render the study of mechanics appalling. Such, for example, is the stiffness of ropes, the flexibility of bars, the friction of the various parts in contact, and the weight of the machine itself.

For the present, therefore, we shall consider all surfaces as being perfectly smooth, ropes and chains as being perfectly flexible and possessing neither thickness nor weight, and all levers as being perfectly rigid.

These conditions are, of course, impossible; nevertheless, if they are assumed, it will be found to materially assist the student in understanding the relation between the power and weight of a machine. This relation once determined on the supposition that none of these detrimental though ever-present resistances exist, it becomes an easy task to correct the result when their effect is afterwards ascertained. Indeed, this is the most expeditious method of arriving at the effective force of any machine.

One of the first problems that naturally arises in the investigation of a machine, is the determination of the *power* which will produce a state of equilibrium with respect to the weight.

Virtual[1] Velocity and Virtual Moment.—The power and weight being connected together through the medium of the machine, any motion communicated to the one will act upon the other. It is found that a certain proportion exists between the motion of the one and the motion of the other, which proportion depends entirely upon the construction of the machine. To bring about equilibrium in the machine, it is necessary that the power should have the same proportion to the weight that the velocity of the weight has to the velocity of the power; in other words—

The power multiplied by its velocity must be equal to the weight multiplied by its velocity.

This movement of the point of application of a force in its own direction, is called its *virtual velocity*; and the product of the virtual velocity by the force, is called the *virtual moment.*

Suppose a weight of 20 pounds to move over 2 feet in

[1] L., *virtualis*, potential.

a given time, its virtual moment is $20 \times 2 = 40$. Now, suppose a weight of 5 pounds to move over 8 feet in the same time; the virtual moment is $5 \times 8 = 40$; hence the two weights are in equilibrium.

In these examples the virtual moments of each of the weights equal 40; but the virtual velocity of the one is 2 feet, and of the other 8 feet in the given time.

From the above example it is evident, that *what is gained in power is lost in speed.* For if by the exertion of 5 pounds we move a weight of 40 pounds, we have to exert that power with eight times the speed that we should have to exert a power of 40 pounds applied direct.

The converse of this theorem is also true, viz., *what is lost in power is gained in speed.* The truth of this fact will become evident in the numerous examples which the student will have presented to him.

CHAPTER V.

THE MECHANICAL POWERS.

Varieties.—In this and succeeding chapters we shall consider those simple machines which constitute the basis of all machinery, whether simple or complex. They are six in number, namely,—1, *The Lever;* 2, *The Wheel and Axle;* 3, *The Pulley;* 4, *The Inclined Plane;* 5, *The Wedge;* and, 6, *The Screw.* These six elements of machinery are extremely simple when compared with the results they bring about. As we proceed we shall be able to further reduce their number; for the principles of some of them are identical, and only differ in appearance. Thus, we shall be able to show that the wheel and axle is a lever, and that the wedge and screw are only modifications of the inclined plane. The simple machines are thus reduced to three, namely, *The Lever, The Pulley,* and *The Inclined Plane.* By means of these any force may be transmitted to any point, to produce any result.

1 (*a*). *THE LEVER.*

Description.—A lever may be defined as a rigid bar, capable of being moved about a fixed point. It is generally used to move heavy bodies, when the force to be applied is comparatively small. A familiar example is seen in a crowbar used to raise a heavy stone. One end of the crowbar is inserted beneath the stone; a smaller stone is placed beneath the crowbar near the large stone; and a man, pressing on the end of the bar, is enabled to raise the heavy stone, which he would otherwise be incapable of moving. The crowbar in this case is a lever; the small stone is the fixed point upon which it is moved; the man's strength is the power applied; and the heavy stone is the weight to be raised. In all levers these three points must be recognised. First, the point upon which the lever turns, called the *Fulcrum*;[1] second, the force applied, called the *Power*; and, third, that which has to be raised, called the *Weight*.

Classification.—Upon the relative positions of the fulcrum, power, and weight, the classification of levers is based. They fall into three groups, and are called respectively levers of the first, second, and third orders.

In levers of the first order, the fulcrum is placed between the power and the weight.

In levers of the second order, the weight is placed between the fulcrum and the power.

In levers of the third order, the power is placed between the fulcrum and the weight.

In fig. 27, I. represents a lever of the first order; II., one of the second order; and III., one of the third order—F in each case representing the fulcrum, P the power, and W the weight.

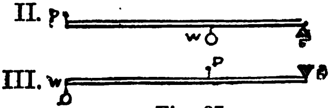

Fig. 27.

[1] L., *fulcrum*, a prop.

Equilibrium of Levers.—To produce equilibrium in a lever, the resultant of the forces acting upon it must pass through the fulcrum.

Fig. 28.

Levers of the First Order.—If P W (fig. 28) be a lever moving freely about its fulcrum, F, P being the power, and W the weight, let us see the conditions under which P will support W, with respect to the principle of virtual velocities.

If the lever be caused to oscillate about the centre F, so that P shall descend at the same time as W ascends, P and W, the extremities of the lever, will move through similar arcs, having F as their common centre, and F P, F W as their respective radii. The arcs will be the spaces through which W and P move; and whatever be the magnitude of the arcs, they will be proportional to the spaces through which W and P move.

Similar arcs are proportional to their radii, hence the descent of P : the ascent of W : : F P : F W. The radii may therefore be considered to represent the motions of the P and W.[1]

Hence, it is clear, that the *power is to the weight, as the distance of the* WEIGHT *from the fulcrum is to the distance of the* POWER *from the fulcrum.* That is to say, if the *power*, multiplied by its distance from the fulcrum, equals the *weight* multiplied by its distance from the fulcrum, the machine is in equilibrium, and the power exactly balances the weight.

To make this clear, let W (fig. 29) represent a weight of 40 pounds suspended at a distance of four

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[1] Let P = the power applied at A; W the weight applied at B, and F the fulcrum. Let the distance A F = A and B F = B. Then, by the principle of parallel forces, the equation to produce equilibrium will be P : A : : W : B.

inches from the fulcrum, F, of the lever; and let P

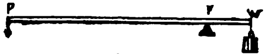

Fig. 29.

=8 pounds, and the distance P F=20 inches. Then $40\times4=160$, and $8\times20=160$; consequently a P of 8 pounds at a distance of 20 inches from F, exactly balances a W of 40 pounds at a distance of 4 inches from F.

From this it appears that the power P, at different distances from F, may be made to balance different weights. This is the case in the common steel-yard, where the substance to be weighed is suspended from the short arm of the lever, and its weight is ascertained by moving another weight along the longer arm, which is graduated.[1]

It is evident that the same effect will be produced if the weight P is fixed, and the fulcrum F is movable. This is the case in the Danish balance (fig. 30), where

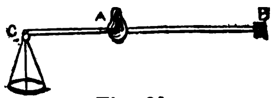

Fig. 30.

the fulcrum is movable along the bar B C, which is graduated; the weight of C being ascertained by the position of A.

We have already shown that the product of a force by the perpendicular from a point in its direction, is called the *moment of that force.*

Hence, in fig. 29, as P=8 lbs. and P F=20 inches, $8\times20=160$ is the moment of the power P. Again, as W=40 lbs. and F W=4 inches, $40\times4=160$; and this is the moment of the weight W. From this it appears,

---

[1] L., *gradus*, a step; marked off in equal divisions.

that *when the moments of two forces about any two points on the opposite sides of a centre are equal, then the two forces are in equilibrium.*

And *if the sum of the moments of the forces which tend to turn a lever round in one direction, be greater or less than the sum of the moments of the forces which tend to turn it round in the other direction, the lever will move in the direction of the greater.*

The *balance* is a lever of the first kind, the arms of which are of equal length. It generally consists of a metallic bar, or *beam*, supported on a knife-edged fulcrum, fixed horizontally under the centre of the beam. Attached at right angles to the beam, is an index which moves over an arc, and stands at zero when the beam is horizontal. Pans for receiving the bodies to be weighed are suspended from the extremities of the beam; and consequently at equal distances from the fulcrum. Equal weights placed in the scales will cause the beam to remain horizontal if the balance be *true;* for the distances being equal between the centre and the two points of suspension at which the weights act, these distances, multiplied by equal weights, give equal moments, and consequently there will be no tendency of the beam to turn one way or the other. If the beam is not horizontal, the balance is *false.*

Now it has been shown that a body can only remain in equilibrium when the C G is either above or below the point of support. When the C G is above, the body is in unstable equilibrium, and, if moved from its position, will not come to a state of rest until the C G has reached the lowest possible point. Hence, if in the balance the C G of the beam be above the point of support, a little force only will be necessary to turn it; but, once moved, it will *not* return again to the same place. The equilibrium must therefore be stable, and this is secured by throwing the C G of the beam and scales *a little below the point of support.*

When unequal weights are placed in the scales, the balance should readily indicate the side on which the heavier weight is placed. This is called the *sensibility* of the balance, and depends on two things. First, when the beam is moved, the C G is thrown out of the vertical; and its tendency to regain that position is in proportion to the weight of the beam. Wherefore the greater the weight of the beam, the greater is the power required to move it from the vertical. *The weight of the beam should therefore be as small as possible, compatible with the necessary strength.* Second, the longer the arms, the greater is the product of their length by an equal weight, or, in other words, the greater is the moment. Hence *the arms should be as long as possible.*

We will now consider what would be the effect if unequal weights are placed in the scales of a beam whose arms are of equal length.

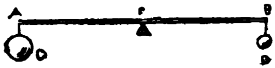

Fig. 31.

Let the arms, A F, B F (fig. 31), of the beam A B be equal. From A and B let weights C and D be suspended; and suppose C to be heavier than D. The tendency of C to depress the arm A F is measured by multiplying the weight C by the length A F. The opposing tendency of D, is D multiplied by B F. Now A F and B F being equal, the product of C with A F will be greater than D with B F; hence C will have a greater tendency to depress A F than D to raise it, consequently A F will descend.

From this we may conclude that, if the balance be true, the beam will remain horizontal only when equal weights are suspended from each arm; and that if either weight be greater than the other, the beam will incline towards the greater weight.

Fraudulent Balances are sometimes constructed with

the arm from which the substance to be weighed is suspended longer than the arm to which the true weight is hung. As the balance can only remain horizontal when the moments of force on either side of the fulcrum are equal, the product of the longer arm by a *smaller weight* will balance the product of the shorter arm by the true weight. To test a balance, it is only necessary to transfer the weights from one scale to the other. If the balance remain in equilibrium, it is true; but if one side preponderates, the balance is false. The true weight of any article in a false balance may be obtained by finding the counterpoise which will bring the beam horizontal after the weights have been transposed. Multiply this counterpoise by the original one, and the square root of the product will be the true weight. Thus, if one counterpoise be four pounds, and the other nine pounds, $4 \times 9 = 36$ the square root of which is six. Six pounds then is the true weight.[1]

Levers of the Second Order.—In a lever of the second order, the weight lies between the fulcrum and the power; and to produce equilibrium, since the power

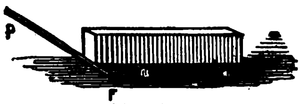

Fig. 32.

is farther from the fulcrum, it must be less than the weight.

A familiar example of the use of a lever of the second kind is shown in the crow-bar used as in fig. 32. The power is applied at P; the fulcrum is at the opposite end of the lever; and the weight occupies an intermediate

[1] Let A and B be arms of a balance. T the true weight of a body, W the weight at B which balances T at A, and X the weight at A which balances T at B. Then $T \times A = W \times B$, and $T \times B = X \times A$. By the multiplication of these equations, we find that $T = \sqrt{W \times X}$.

position. In the oar of a boat we have another example. The water here is the fulcrum, the boat acting at the rowlock is the weight, and the power is applied at the

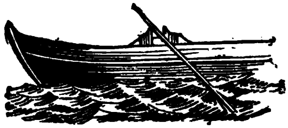

Fig. 33.

handle of the oar. Water, however, being a liquid, the fulcrum is movable, and a loss of power is thereby occasioned.

A wheel-barrow affords a third illustration of a lever of the second order.

Levers of the Third Order.—In these levers the power is nearer the fulcrum than the weight is, it must therefore be greater than the weight, to produce equilibrium. There is, consequently, an apparent mechanical disadvantage in levers of this kind, since the force applied to raise the weight is greater than the weight itself. As the velocity of the weight, however, bears the same ratio to the velocity of the power, as the distance of the weight from the fulcrum bears to the distance of the power from the fulcrum, a much greater velocity is imparted to the weight than would be attained through the intervention of levers of any other order.

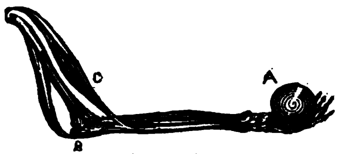

Fig. 34.

This kind of lever is only used where velocity is the object to be attained, rather than an economy of force. The human arm (fig. 34) is an example of this order of

lever, in which B is the fulcrum, C the power, and A the weight. Great force is exerted by the muscle C through a small distance; and the weight A is thereby raised through a greater distance and with a greater velocity than could otherwise be obtained.

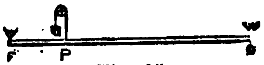

Fig. 35.

Let fig. 35 represent a lever of the third order, in which $FP = 6$ inches, $FW = 18$ inches, $P = 6$ lbs., and $W = 2$ lbs. The products of $FP \times P = 6 \times 6 = 36$; and $FW \times W = 18 \times 2 = 36$.

The moments about P and W are, therefore, equal; but the velocity of W is three times that of P.

Pressures on Fulcra.—It will be evident that in a lever of the first order the power and weight act in the same direction, while in levers of the second and third order they act in opposite directions. The pressure upon the fulcrum, therefore, in levers of the first order is equal to the sum of the power and weight; and in levers of the second and third order it is equal to the difference between the power and weight.

CHAPTER VI.

THE MECHANICAL POWERS (*continued*).

1 (*b*). *MISCELLANEOUS LEVERS.*

Compound Levers.—When the power is transmitted through the medium of a series of levers, instead of a single lever, the machine is called a compound lever.

Power.—The power obtained by this lever is calculated in the same way as in a simple lever.

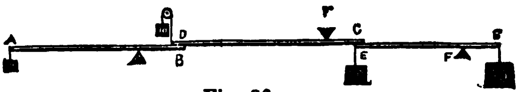

Fig. 36.

Thus, suppose (fig. 36) each of the longer arms to have twice the length of the shorter ones; then 5 lbs. at A will support 10 lbs. at B; and B pressing upwards on D with a force of 10 lbs., C presses downwards on E with a force of 20 lbs., which counterbalances 40 lbs. at G. Consequently a pressure of 5 lbs. on A, through the media of 3 levers, counterpoises 40 lbs. at G.

Hence, *the power is to the weight as the continued product of the shorter arms is to the continued product of the longer arms.*

Supported Bars.—If a weight be suspended from a bar resting on two supports, the bar becomes a lever. Let A B (fig. 37) be a bar resting on the two supports

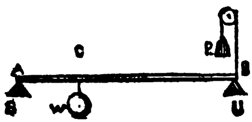

Fig. 37.

S and U, and W the weight supported by the bar. Remove the support U, and let the weight P be substituted for it; P will therefore equal the pressure upon U, and A B becomes a lever of the second order.

By the rule above established, the power multiplied by its distance from the fulcrum, equals the weight multiplied by its distance from the fulcrum. The supports, therefore, divide the weight W between them in the *inverse proportion of their distance from it.*

Thus, if in fig. 37 A C=4 inches, C B=12 inches, and W=60 lbs, A will bear 45 lbs. and B 15, since B C is three times A C.

Curved Levers.—Levers are sometimes curved, as in fig. 38, where A and B are the points of application of

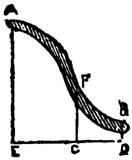

Fig. 38.

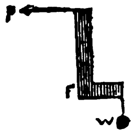

Fig. 39.

the Power and Weight and F the fulcrum. The distance through which the power A and weight B act are the perpendiculars E C, D C, drawn from the fulcrum in the direction of the power and weight.

Angular Levers.—Another form of lever in frequent use, is the *angular lever*, fig. 39. It acts in precisely the same manner as the ordinary lever—that is to say, the power is to the weight as the distance W F is to P F. A hammer with a claw used for drawing a nail, is a lever of this kind. F P being the handle, F W the claw, P the force used in drawing, and W the resistance of the nail.

Angular levers are used where it is desirable to cause the weight to move in a different direction from that in which it would move if the lever were straight.

Mechanical Efficiency.—The ratio of the weight to the power is called the *mechanical efficiency* of a machine; or, in other words, *the weight divided by the power equals the mechanical efficiency.* Thus, if the weight be fifteen times the power, the mechanical efficiency is said to be 15.

Any lever may have its mechanical efficiency varied at pleasure; for by altering the position of the power or weight with respect to the fulcrum, the same lever

can be made to act with greater or less advantage, or even at a disadvantage.

Weight of Levers.—The weight of the lever itself is concentrated at its centre of gravity, and increases or diminishes the power according to the position of the centre of gravity with respect to the fulcrum.

Rules.—The following rules for the practical application of the lever are useful.

LEVERS OF THE FIRST ORDER.—Divide the weight by the power; and the quotient will be the difference of leverage, or the distance from the fulcrum in terms of the short arm at which P will equal W. Or the product of W × distance from F, is to the product of P × distance from F, as W : P.

Examples.—(1) A weight of 2000 lbs. is to be raised by a power of 80 lbs., one arm of the lever being 2 feet, what is the length of the long arm? $\frac{2000\times 2}{80}=50$ feet. Or $2000\times 2=4000$ and $80\times 50=4000$.

(2) A weight of 2460 lbs. is raised by a 7-foot lever and 300 lbs., where is the fulcrum? $\frac{2460}{300}=8{\cdot}2$. Or long arm : short arm :: 8·2 : 1, therefore $\frac{7\times 12}{8{\cdot}2+1}=\frac{84}{9{\cdot}2}=9{\cdot}13$ inches = the shorter arm.

LEVERS OF THE SECOND AND THIRD ORDER.—As the distance between P (or *W*) and F : the distance between the W (or *P*) and F :: the effect to the P or the P to the effect.

Examples.—(1) What P will raise 1500 lbs., W being 5 feet from P and 2 feet from F?

$$5+2=7 : 2 :: 1500 : 428{\cdot}5714=\text{P}.$$

(2) What W is supported by each end of a beam 30 feet long, resting on blocks and having 6000 lbs. placed 10 feet from one end?

30 : 20 :: 6000 : 4000, at the end nearest W,

and 30 : 10 :: 6000 : 2000, at the end farthest from W.

CHAPTER VII.

THE MECHANICAL POWERS (*continued*).

2 (*a*). *THE WHEEL AND AXLE.*

Introduction.—The wheel and axle is only another form of the lever. It is used when the distance through which the weight has to be raised is very great, as in the case of a bucket from a well.

When the distance through which the weight has to move is very small, the lever is admirably adapted to produce the effect. But if it were required to raise the weight through a considerable distance by means of an ordinary lever, either the arm would have to be inconveniently long, or the weight would have to be raised by successive efforts with a shorter lever, which would necessitate the raising of the fulcrum after each effort. In order to avoid this intermittent motion, the lever is modified into the machine called the *wheel and axle.*

Description.—This machine consists of two cylinders,

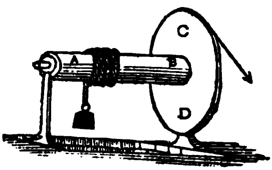

Fig. 40.

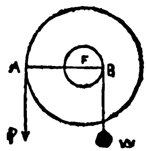

Fig. 41.

A B and C D (fig. 40), having a common axis, but differing in radius. The smaller cylinder is called the *axle*, and the larger the *wheel.*

The Power is applied at the circumference of the *wheel*, and the *Weight* acts on the circumference of the *axle.* Round the cylinder is wrapped a rope, to which the weight to be raised is attached.

Reference to fig. 41 will convince us that this machine is only a *perpetual lever.* In this figure F is

the centre of the axle, F B the radius of the axle, and F A the radius of the wheel. The power acts at A, the extremity of the long arm A F; and the weight acts at B, the extremity of the shorter arm B F. Consequently, P : W :: A F : B F. In other words, *the power is to the weight as the radius of the axle is to the radius of the wheel.*

Virtual Velocity.—It is easy to perceive that the power descends in one revolution of the wheel through a space equal to the circumference of the wheel, and the weight ascends a space equal to the circumference of the axle. The virtual velocities of the power and weight are therefore equal—that is to say, *the power multiplied by its velocity equals the weight multiplied by its velocity.*[1]

The force is conveyed to the axle in a great variety of ways, of which a few of the principal will be now described.

Windlass.[2]—The windlass, is a modification of the wheel and axle used for raising heavy weights—such as a ship's anchor. Instead of a wheel, levers called *hand-spikes* are inserted in holes cut in the axle. In this case means must be used to prevent the reaction of the weight during the removal of the levers. Such a contrivance is represented in fig. 42, and is called a *ratchet wheel.*[3]

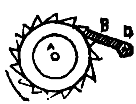

Fig. 42.

A is the ratchet-wheel having teeth carved in the direction on which the rope is coiled round the cylinder. B is a bolt called a *paul,* movable on its centre, D, and by its weight falling between the teeth of the wheel A.

[1] In the wheel and axle we have two parallel forces, P and W, acting at the ends of the lever A B moving on its fulcrum F. The equation to produce the equilibrium is therefore $P \times AF = W \times BF$, if the power acts in the direction of a tangent, as in figure 40. Hence, if X be the point on the circumference at which the power acts, $X \times F$ is the moment of X about F. But $XF = AF$, therefore, $X \times F = W \times BF$.

[2] Dutch, *winden,* to wind; *as,* an axis. [3] Ital., *rochetto,* a spindle.

From the direction of the teeth, it is evident that the wheel can be moved round only in the direction of the arrow; any tendency to revolve in the opposite direction being checked by the paul B falling between the teeth, immediately the power is withdrawn.

Capstan.—The axle is sometimes placed vertically; the machine is then called a *capstan*,[1] and possesses

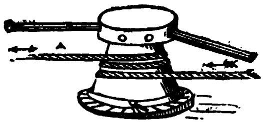

Fig. 43.

considerable advantages over the ordinary windlass. The levers are inserted into holes in the capstan head, and the men walk round, pushing the levers before them. Any number of men may thus exert their strength upon the axle; and the power is constant, which is not the case with the horizontal axis, as will be shown immediately.

The barrel of the capstan is always of a conical form. The rope is wound round the barrel till the end is reached. The part A is then slackened, or *surged*, and the coils slip up towards the narrow end of the barrel, and a second series is then wound on. The number of coils wound on to the capstan barrel to start with, depends upon the strain, three being generally sufficient to cause the rope to so bite the capstan that it may be held tight by a boy at A.

Winch.—In another case the axle is terminated by a bent lever, called a *winch*, by means of which the axle can be turned by the workman.

In this case the power is not uniform. By referring to the figure it will be seen that when, in the revolu-

[1] L., *capistrum*, a halter; so called from being used with a rope.

tion of the handle, the operator is exerting his strength from A to B and C, his weight will assist him, but in

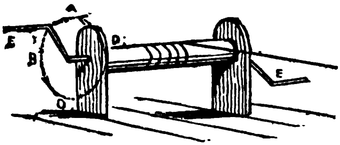

Fig. 44.

the opposite part of the revolution, from C through D to A, he has to raise his body as well as the handle. To equalize this motion as much as possible, a double winch is used, with handles fixed in opposite directions, so that one descends while the other ascends. The capstan has not this disadvantage, and is therefore superior to the winch, where a number of men are to act upon a single axis.

Steering Wheel.—In a ship's steering wheel, levers as spokes project from the rim of the wheel.

Mechanical Efficiency.—The mechanical efficiency of the wheel and axle, depending theoretically upon the ratio existing between the radii of the wheel and of the axle, may be increased or diminished by increasing or diminishing that ratio, that is by enlarging the wheel or by diminishing the diameter of the axle. These alterations, however, can only be carried to a certain extent. For it is evident that if the wheel be too large, the power will have to act through an inconveniently large space. Again, if the axle be very small, its strength will be reduced so that the machine can only be employed for the raising of small weights.

It is necessary also, in constructing the machine, to consider the *direction* of the axis. If it be horizontal, and the centre of gravity of the whole machine fall midway between the two pivots supporting it and the wheel, the weight of the machine will press equally upon both. If the axis be vertical, which is sometimes

a convenient position, the whole weight will rest upon the lowest pivot, which should therefore be correspondingly strengthened.

Omitted Resistance.—In order to obtain the real mechanical efficiency of the wheel and axle, deductions have to be made from its theoretical efficiency, on account of the stiffness, weight, and thickness of the rope. Its stiffness and weight are real additions to the weight to be raised; while its thickness, by increasing the radius of the axle (the effect of the rope being considered to act through its centre), diminishes the ratio between it and the radius of the wheel. In addition to these, the weight of the machine and friction have to be allowed for.

Differential Wheel and Axle, or Chinese Windlass.—In cases where great resistance has to be overcome, the difficulty of so proportioning the ratio of the power to the weight, as to make the machine efficient without reducing the strength of the axle or increasing the radius of the wheel too much, is overcome by a contrivance called the *differential wheel and axle, or Chinese windlass.*

Fig. 45 represents this machine. A B is the axle,

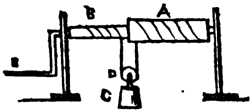

Fig. 45.

consisting of two parts of different diameters. D is a pulley to which the weight C is attached. The rope is rove through this pulley, and coiled on both the thick and thin parts of the axle in the same direction. By turning the handle of the winch E, the rope is coiled on the thicker part, and wound off the thinner one.

Every revolution of the axle will draw up a length of rope equal to the circumference of the thick axle, and at the same time a length equal to the circumference of the thin axle will be let down.

In a single revolution the length of the hanging part of the rope is diminished by a length equal to the difference between the circumferences of the large and small parts of the axle. The weight is lifted by means of the pulley D, which is suspended by equal lengths of rope. The distance through which the pulley, and consequently the weight, will be raised by each revolution, is therefore *half* the length by which the rope is diminished.

Equilibrium will therefore be established when *the power, multiplied by the length of the lever which turns the axle, is equal to the weight multiplied by half the difference of the radii of the two parts of the axle.*

One advantage in this machine is, that there is no reaction of the weight when the power is withdrawn. On the other hand, a large amount of rope must be wound up to raise the weight even a small distance; though less than is requisite in any other machine of the same efficiency, in which the power is transmitted through ropes.

Fusee.—When a power which varies in intensity is required to produce uniform motion in the weight, it is effected by adopting this principle of various diameters in the cylinder, or, in other words, by applying different leverages to the weight. This is well illustrated in the *fusee*[1] of a watch (fig. 46). A is a *drum* containing a

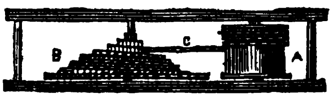

Fig. 46.

spiral spring of finely tempered steel, fixed at one end on an axis. The other end is attached to the inside of the drum A, which freely revolves round the axis. A chain, C, is coiled round the drum; one end being fixed

[1] L., *fusus*, a spindle; F., *fuseau*.

to the drum, and the other to the lower part of the fusee B. This fusee is a conical figure turning on an axis, and on it a spiral groove is cut to receive the chain C.

Let us suppose the watch to be wound up. All the chain will then be wound off the drum A on to the fusee B, the last coil being at the small end of the fusee; and the spring in A will be stretched to its utmost intensity. Its tendency to recoil will be then at its greatest, and will diminish in proportion to the extent of its recoil. Now, if the chain were attached to a cylinder, the motion imparted to it by the spring would be a gradually diminishing one, the watch would consequently lose time. But the chain being wound on a cone, when the pulling force of the spring is at its greatest, the leverage on the axis of the fusee is at its least; and as the strength of the spring diminishes, so the leverage on the axis of the fusee increases. These are so adjusted as to result in a motion of the axis of the fusee at a uniform rate.

CHAPTER VIII.

THE MECHANICAL POWERS (*continued*).

2 (*b*). *COMPOUND WHEEL WORK.*

Relations with Levers.—As it is sometimes convenient to adopt a system of levers to raise a weight, so it is occasionally desirable to produce a certain effect by a system of wheels, called *a train of wheels.* The wheel and axle having been shown to be a modification of the lever, so a *system of wheels* is only another form of compound lever; and the conditions necessary to bring about equilibrium are the same in both.

As in the compound lever the power is applied to the longer arm of the lever, which acts through the short arm on the long arm of the next lever; so in complex wheelwork the power is applied to the *circumference* of the first wheel, which transmits its effect through its own

axle to the *circumference* of the next wheel. This is effected by various means.

Friction Wheels.—Where the weight is very small, the mere friction of the wheels in contact is sufficient to impart motion, as in fig. 47, where the power is

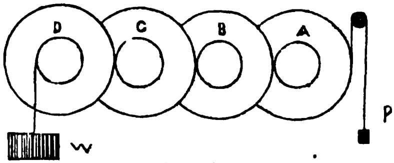

Fig. 47.

applied to the circumference of the wheel A, and is transmitted through its axle to the circumference of the second wheel, B, and so on. Each separate wheel and axle being a lever, the effect of the above combination is the same as in a system of levers the longer arms of which are equal to the radii of the wheels, and the shorter arms equal to the radii of the axles. Equilibrium will therefore be produced when *the product of the power by the radii of all the wheels is equal to the product of the weight by the radii all the axles.*

Pulleys, or Drums.—Another method of transmitting the power of the axle to the succeeding circumference, is by straps or cords, G G, H H (fig. 48), passing over

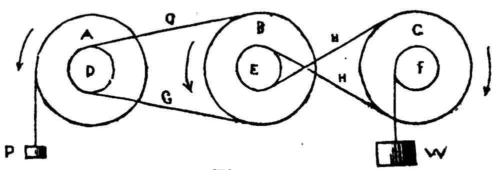

Fig. 48.

the circumference of the wheel and axle; and being drawn tight, the *tension*,[1] or strain, will generate sufficien

[1] L., *tendo*, to stretch.

friction between the rope or band and the circumferences to transmit the power from one to the other. The wheel is called a *drum* when a strap is used, and a *pulley* when a cord is used.

When it is intended that both the driving wheel and the wheel to which it communicates motion should revolve in the same direction, the cord is merely carried round both circumferences, as in G G. But when it is intended that the wheels shall revolve in opposite directions, the strap is crossed, as H H. This latter method possesses the advantage of having more surface of the circumference to act upon, and consequently the friction, or *bite*, is greater.

Cog Wheels.—The most common method, however, of transmitting this force, is by means of *cog-wheels*, that

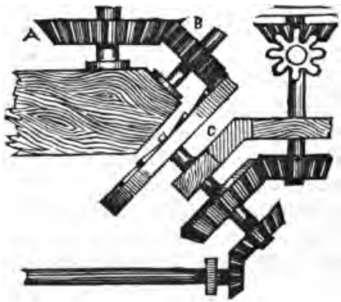

Fig. 49.

is, wheels with projections standing out from their circumferences, which are called *webs*. The projections on the webs of the wheel are called *cogs, or teeth;* and those on the web of the axle are termed *leaves;* and the axle itself is generally called a *pinion*.[1]

Several conditions must be fulfilled in each pair of wheels, to ensure their working together. The teeth of

[1] L., *pinna*, a feather. The leaves being compared to feathers.

each wheel must be equal and equidistant; and the teeth of one wheel must be equal to and equidistant from those of the other. The number of teeth in each wheel will therefore be proportional to their circumferences. Consequently, in speaking of the ratio of one wheel to another, we may either say that one is a wheel of 72 teeth, and the other one of 12, or that one is six times larger than the other. *The power multiplied by the teeth in the wheels, equals the weight multiplied by the leaves in the pinions, when the machine is in equilibrium.* The circle midway between the grooves and summits of the teeth, is called the *pitch circle;* and the motion transmitted by the contact of the teeth is the same as would be produced by the rolling contact of the pitch circles.[1]

Hunting Cog.—In determining the number of teeth in a wheel and the pinion which works with it, it is desirable that the same pair of teeth should come in contact as seldom as possible, in order to lessen the friction, and consequent wear and tear. For example, if a wheel has 80 teeth, and its pinion 10, it is evident that every tenth tooth of the wheel would *engage* with the same leaf of the pinion, and that each leaf of the pinion would come in contact with the same eight teeth in every revolution of the wheel.

Were the teeth mathematically exact, this would be of little consequence. But as the best-cut wheels vary slightly in their teeth, these inequalities are compensated for by making teeth and leaves so work that each leaf

---

[1] *Equation of equilibrium for a pair of toothed wheels.*—Let the wheels be in equilibrium with a weight, A, at the axle of the larger, and a weight, B, at the axle of the smaller, and let R be the pitch circle of the larger wheel and S that of the smaller. Let the radii of the axles be equal and represented by r, and let the reaction of the wheels equal X. Then the equilibrium of the large wheel gives $A \times r = X \times R$, and of the smaller, $B \times r = X \times S$.

$$\text{Wherefore } \frac{A}{B} = \frac{R}{S} = \frac{\text{No. of teeth in the large wheel.}}{\text{No. of teeth in the small wheel.}}$$

shall come in contact with every tooth of the wheel in succession before it engages with the same tooth a second time. This is effected by making the number of teeth and the number of leaves such that no integer will divide them exactly; or, arithmetically speaking, by making them prime to each other. Thus the number of the teeth is made just one more than a number exactly divisible by the number of leaves. The extra cog is called the *hunting cog*. Suppose, for instance, there are eight leaves, and the diameter of the wheel is required to be about eight times the diameter of the pinion. If the wheel had 64 teeth, it would be exactly eight times the diameter of the pinion; but the teeth and leaves would not *hunt*. By cutting the wheel with 65 teeth, and supposing the leaf A to be first engaged with the tooth B, it will, after eight revolutions of the pinion, be engaged with the tooth before B, and after eight more revolutions of the pinion it will be engaged with the second tooth before B. Hence every revolution of the wheel is $\frac{1}{65}$ of a revolution behind, and the pinion must revolve 8×65, or 520 times before the leaf A again engages with the tooth B. Hunting cogs, however, are only necessary where the strain on the cogs is great, as in mill-work.

Lantern.—Sometimes the pinion is furnished with a *lantern* (fig. 50), A, instead of ordinary teeth. This

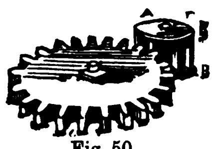

Fig. 50.

consists of two circular discs, B B, with cylindrical teeth between them. This form of pinion is used where the pressure is so great that ordinary leaves would be in danger of breaking.

Rack and Pinion.—If a small wheel with cogs be caused to work with a bar, B (fig. 51), having cogs the same distance apart as those of the wheel, the motion of A round its axis will cause B to move longitudinally. This is called a rack[1] and pinion. It is evident in this case that the rack B is analogous to the rope in the wheel and axle.

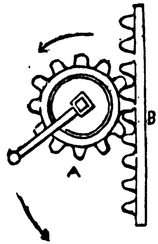

Fig. 51.

To find the Mechanical Efficiency of a Crane.—Divide the number of the driven teeth by the number of the drivers, and the quotient will be the relative velocity, which, multiplied by the length of the winch and force in pounds, and divided by the radius of the barrel, will give the weight the crane is capable of raising.

Example.—A force of 18 lbs. is applied to a winch, which is 8 inches long; the pinion has 6 teeth, and the wheel 72; and the barrel is 6 inches in diameter. What weight will be raised?

$$\frac{72}{6} = 12 \times 8 \times 18 = 1728 \div 3 = 576 \text{ lbs.}$$

If w = winch, r = radius of barrel, P = power, v = velocity, and W = weight; then

$$W r = v w P \therefore W = \frac{v w P}{r} \text{ and } P = \frac{W r}{v w}.$$

CHAPTER IX.

THE MECHANICAL POWERS (*continued*).

3. *THE PULLEY.*

Preliminary.—When we wish to apply force to a body, we can do so by the direct application of the hand; by pushing or pulling with a stick; or by pulling a rope attached to the body. The transmission of the power through the stick is due to its *rigidity*.[2] The transmission of the power through the rope is due to

[1] A.-Sax., *ræccan*, to reach, extend. [2] L., *rigidus*, stiff.

its *inextensibility*.[1] A pulling force, therefore, applied to one end of a rope, has an equal effect at the other end. Owing also to the *flexibility*[2] of a rope, this force in one direction may be made to balance an equal force in any other direction. Thus, if we have a power acting in the direction A P (fig. 52), it can be made to exert an equal power in the direction W A, by passing

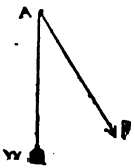

Fig. 52.

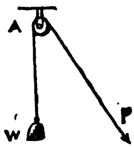

Fig. 53.

the rope over a fixed point, A, situated at the point of junction of the lines P A and W A. Any appreciable excess of P over W, will cause the rope to move towards P. The point A being fixed, and the rope flexible and inextensible, if P moves any distance in the direction A P, W will move an equal distance in the direction W A.

A force applied by means of a cord is called *tension*.[3]

Definitions and Description.—As no rope is perfectly flexible, the friction in passing over a fixed object at A, would be so great as to absorb a great portion of the power. It is usual, therefore, to pass the rope over a wheel, A (fig. 53), turning freely on an axle. This wheel and axle is called a *pulley*. The wheel, usually called the *sheave*, has a groove cut on its circumference, to allow the rope to pass over it without slipping off. The wheel turns freely on an axle, or *pin*, within a case called the *shell*; and the whole apparatus of shell, pin, and sheave, is called a *block*.

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[1] L., *in*, not; and *extensibilis*, able to be stretched. [2] L., *flexibilitas*, from *flecto*, to bend. [3] L., *tensus*, stretched.

The rope or iron band passing round the block, with a hook attached, for the purpose of fixing the block in any required position, is called the *strap*, or *strop*.

Varieties of Blocks.—Blocks are of various kinds. When they contain one sheave (fig. 54), they are called

Fig. 54.

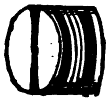

Fig. 55.

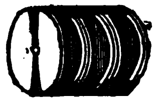

Fig. 56.

single blocks; when two sheaves (fig. 55), *double blocks;* when three sheaves (fig. 56), *threefold blocks*, and so on. Large blocks are called *purchase blocks*.

Fig. 57

Fig. 58.

Fig. 59.

Sometimes blocks have one sheave above the other. When the sheaves are equal in size, the block is called a *sister block* (fig. 57); when one is larger than the other, it is called a *fiddle block* (fig. 58); and when the plane of one sheave is in a direction contrary to that of the other, it is called a *shoe block* (fig. 59).

The Rope.—The rope passing over the sheave is called the *fall*. The part of the fall to which the power is applied, is called the *hauling part;* and the part to which the weight is applied, is called the *standing* part. A system of blocks is called a *tackle;* and when the blocks are very large, and are used for the purpose of raising heavy weights, the system is called a *purchase*.

When chain is used instead of rope, the block over which it passes is called a *gin*.

Use of the Pulley.—By means of the pulley and flexible cord, we are enabled to balance a force exerted in one direction by an equal force exerted in the same or another direction. The pulley is only made use of to change the direction of the power, and may be considered to be a lever of the first kind. Thus, fig. 60 is a section of the sheave of a pulley, in which A

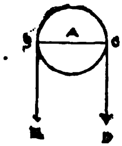

Fig. 60.

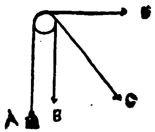

Fig. 61.

is the axle or pin. A force, D, in the direction C D, acting on the arm A C of the lever B A C, is made to balance a force, E, acting in the direction E B, applied to the arm B of the lever. These arms being equal in length, equal forces applied to the arms will produce equilibrium. It is evident, therefore, that *there is no power gained by the pulley ITSELF.*

The rope possessing inextensibility, the power necessary to balance a weight, is the same, whether the two parts of the rope are parallel, as A B (fig. 61), or divergent, as A C, A D. Consequently the tension of every part is equal.

Omitted Resistances.—To ascertain the mechanical efficiency of a system of pulleys, it will be convenient to consider the ropes to be perfectly flexible, to have neither weight nor thickness, and the sheaves to move without friction (see p. 44).

To estimate the *working* efficiency of any system, the above detracting forces must be added to the power necessary to raise a given weight.

Fixed Pulleys.—Pulleys are either *fixed* or *movable.* In the use of fixed pulleys there is no advantage, since the power and weight are necessarily equal, but they

are convenient for altering the direction of the power. Thus, by means of a fixed pulley, and a rope passing over it, we can raise the weight by exerting a *downward* force. This arrangement is called a *whip* (fig. 62), and a force of 5 lbs. at B requires an equal force at A to balance it. It is the most common form of pulley and fall used, as may be seen in the crane, window blind, etc. It is evident that the strap of the block C bears a strain of 10 lbs; that is, a weight equal to the sum of the power and weight. Ignorance of this fact has led to many accidents, for a block calculated to bear a strain of one ton, will only be strong enough to *lift* half a ton. In theory, also, the fall need only be of such strength as will bear a little more than the power; but in practice the friction is found to be so great, that the hauling part of the fall has to sustain a considerably greater strain than the standing part; and this proportion increases with the number of the sheaves.

Fig. 62.

Movable Pulleys.—In the case of the movable pulley (fig. 63), the standing part, A, bears half the

Fig. 63.

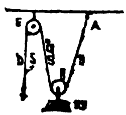

Fig. 64.

weight of B; and consequently a power of 5 lbs. at C, will balance a weight of 10 lbs. at B. If the part C is rove through another fixed block, E (fig. 64), the part C bears a strain of 5 lbs., and so must the part D; and the only advantage of the fixed block, E, is that of changing an upward pull into a downward one. The weight 10, may be considered to be the resultant of the two equal parallel forces, A and C, and is therefore equal to their sum; and $10 \div 2 = 5$, the strain of each point, A and C.

The combination of one whip with another (fig. 65), by connecting the fall of one whip, A B, with the block

Fig. 65.

Fig. 66.

Fig. 67.

Fig. 68.

of another, F, is called a *double whip*, or *whip and runner;* the first whip in this case being called the *runner*. By this arrangement a force of 1 lb. applied to C, will balance a weight of 2 lbs. applied to E.

The standing part, D, of the whip, C D, bears a strain of one pound; hence, if we attach it to the weight, as in fig. 66, we gain an additional lifting force equal to the power. This arrangement is called a *burton*, and the weight is to the power as 3 to 1.

Another method of applying power by means of two single blocks, is the *gun tackle* (fig. 67). In this arrangement it is evident that a power of 1 lb. applied at A, will balance a weight of 3 lbs. applied at B, but only 2 lbs. at C. In all systems of pulleys, it is an advantage to apply the movable block to the weight; and in estimating the working efficiency, the weight of the movable block must be added to the weight to be raised.

A system consisting of a double and single block is called a *luff tackle* (fig. 68). Where the double block is movable, a power of 1 lb. applied to the hauling part will balance a weight of 4 lbs. applied to the block 4. Fig. 69 consists of two double blocks, and is called a *two-fold purchase:* the power is to the weight as 1 to 5.[1]

[1] In figs. 67, 68, and 69, an advantage is gained by attaching the upper blocks to the weight. In Fig. 69, for example, if the weight is fixed to the upper block, the power is to the weight as 1 to 5; but if the lower block is attached, the proportion is as 1 to 4.

Long Tackle.—In blocks having the sheaves side by side, as double and treble blocks, it is difficult in heavy strains to prevent the blocks from tilting or *canting* on one side (fig. 70), owing to the power not being applied in a straight line with the centres

Fig. 69.

Fig. 70.

Fig. 71.

of the blocks. This endangers the splitting of the shells, besides causing additional friction of the sheaves against the sides of the shells. In order to obviate this danger, it is common to cross the fall in reeving it. This brings the standing and hauling parts in a line with the centres of the blocks; but the friction of the one part of the fall over the other is very great.

By reeving the fall through fiddle blocks (fig. 71), all the parts of the fall are in a line with the power and weight, and the larger size of the outer blocks keeps the parts of the fall clear of each other. This arrangement is called a *long tackle.* The only disadvantage here is, that when a number of blocks must be used, and the upper block can only be fixed at a limited height, the two blocks may come together before the weight is raised high enough.

Smeaton's Block.—Mr. Smeaton employed an ingenious invention when building the Eddystone lighthouse, to remedy this evil. A and B (fig. 72), are threefold fiddle-blocks. The fall is rove by fastening the standing part to the hook of the upper block 1, and reeving it through the sheaves 2, 3, etc., the haul-

ing part coming out at 13 on the upper block. In this purchase the power is one-twelfth the weight.

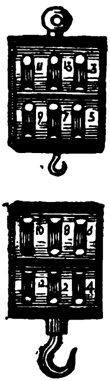

Fig. 72. Smeaton's Block.

Jones's Block.—The great objection to the use of Smeaton's block is, that it requires a combination of at least twelve sheaves, and is therefore only adapted for very heavy work. The accompanying arrangement (fig. 73) can be applied to any number of sheaves from four upwards. The upper block, A, is attached to another block, B, whose sheave is at right angles to that of A; that is, A B is a *shoe block.* The lower block, C, has also two sheaves which diverge from each other, to allow the lower sheave of the shoe block to be of moderate dimensions. To reeve the fall, begin with the upper sheave of the shoe block, and reeve through one of the sheaves of the lower block, then across the lower sheave of the shoe block, and through the opposite sheave of the lower block, and fasten the standing part to the shoe block.

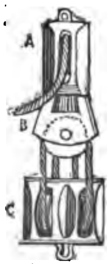

Fig. 73. Jones's Block.

Spanish Burton.—The mechanical advantage of a system of pulleys may be greatly augmented by increasing the number of falls, the standing parts of which are connected with the movable blocks. This arrangement is called a *Spanish burton* (figs. 74–76). The tension of the part A equals the power; and the standing part B being attached to the weight, supports a part of it equal to the power. The hauling part, D, of the fall E C D, supports a weight equal to the tensions of A and B united, and the tensions of C and E being equal, the power is to the weight as 1 to 5.

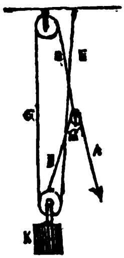

Fig. 74.

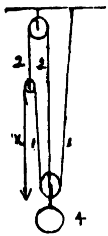

Fig. 75.

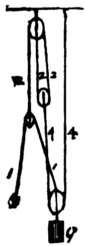

Fig. 76.

Spanish Burton.

Another burton is represented in fig. 75; but by analysis of the application of the power, it will be seen that the weight is only four times the power. In both these systems the weight of the block of the whip assists the power. Thus, in fig. 74, if the weight of G be half that of K, it will exactly balance it; but in fig. 75, the weight of A must be equal to that of B to balance it.

A powerful burton, having an efficiency of 9 to 1, is shown in fig. 76, the figures on each part of the rope showing the multiple of the power borne by each part.

Differential Sheaves.—It would appear that we possess unlimited means of increasing the difference

between the power and the weight, by increasing the number of sheaves in the movable blocks. It is found in practice, however, that the friction of the sheaves and the stiffness of so many parts of the rope is so great, that the power is not by any means proportional to the number of sheaves. Besides which the sheaves are liable to great inequality of wear, owing to their revolving on their pins with different velocities.

Suppose (fig. 72) the block B moves one foot nearer the block A. The standing part of the fall, 2, will be shortened one foot. But the other parts of the fall 2 3, 3 4, etc., will also be shortened one foot. Consequently, while one foot of rope passes over the sheave 1, two feet pass over 2, three feet over 3, and so on.

Now, one foot of rope passing over sheave 1, will move over one foot of its circumference. If the circumference be one foot, it will then revolve once for every foot of rope passing over it; sheave 2 will revolve twice; 3, three times; and 13, thirteen times to every revolution of sheave 1. The wear of each succeeding sheave will therefore increase as the numbers 1, 2, 3, 4, etc. If the sheaves were fixed on their pins so that all turned with the same velocity, another kind of friction would arise; for since various lengths of rope would pass over the different sheaves, the rope would scrape over all the sheaves except that whose circumference coincided with the length of rope passing over it.

If, therefore, we could invent such a system of sheaves ... allow them to revolve on their axles in the ... so as to have equal wear, and also permit ... ferences to pass over spaces equal to the ... passing over the respective sheaves, the ... would be overcome.

Fig. 78. Jones's Blo... Pulley.—An invention of M... (fig. 77). In ... that the ... block, and ... block, and fa...

... be desir...

are exactly equal to the lengths of rope that pass over them. On referring to fig. 72, it is evident that to raise the lower block one foot, one foot of rope must pass round sheave 2, three feet round 4, five round 6, and so on. At the same time, two feet must pass over sheave 3, four over 5, and six over 7. In other words, the rope passing over the lower sheaves is as the numbers 1, 3, 5, 7, 9, etc.; and the rope passing over the upper sheaves is as the numbers 2, 4, 6, 8, 10, etc. Now, circles are to one another as their diameters; and therefore by constructing the sheaves with diameters proportional to the successive integers 1, 2, 3, 4, etc., the revolution of each sheave will be exactly equal to the quantity of rope passing over it. If the sheaves were separate, we should still have the friction of each sheave with the shell to contend with; but by forming grooves having the diameters of the respective sheaves, upon one large sheave, we overcome this difficulty; for the friction will be reduced to that of one large sheave. The expense of construction is the [illegible] obstacle preventing these admirable sheaves from coming into more general use.

Fig. [illegible]
White's [illegible]

Since the fall passes over all the sheaves in [illegible]sion, each part having the same tension, and the [illegible] of the first part being equal to the power, the weight is as many times the power as the number of parts of the fall [illegible] in the movable block [illegible] the standing part. [illegible] the figure of the luff-tackle (fig. [illegible]), the weight [illegible] power [illegible] there are 4 parts of the

between the power and the weight, by increasing the number of sheaves in the movable blocks. It is found in practice, however, that the friction of the sheaves and the stiffness of so many parts of the rope is so great, that the power is not by any means proportional to the number of sheaves. Besides which the sheaves are liable to great inequality of wear, owing to their revolving on their pins with different velocities.

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If, therefore, we could invent such a system of sheaves as would allow them to revolve on their axles in the same time, so as to have equal wear, and also permit their circumferences to pass over spaces equal to the length of rope passing over the respective sheaves, these difficulties would be overcome.

White's Pulley.—An invention of Mr. White supplied this desideratum[1] (fig. 77). In this block the sheaves are so proportioned that their circumferences

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[1] L., *desideratum*, to be desired.

are exactly equal to the lengths of rope that pass over them. On referring to fig. 72, it is evident that to raise the lower block one foot, one foot of rope must pass round sheave 2, three feet round 4, five round 6, and so on. At the same time, two feet must pass over sheave 3, four over 5, and six over 7. In other words, the rope passing over the lower sheaves is as the numbers 1, 3, 5, 7, 9, etc.; and the rope passing over the upper sheaves is as the numbers 2, 4, 6, 8, 10, etc. Now, circles are to one another as their diameters; and therefore by constructing the sheaves with diameters proportional to the successive integers 1, 2, 3, 4, etc., the revolution of each sheave will be exactly equal to the quantity of rope passing over it. If the sheaves were separate, we should still have the friction of each sheave with the shell to contend with; but by forming grooves having the diameters of the respective sheaves, upon one large sheave, we overcome this difficulty; for the friction will be reduced to that of one large sheave. The expense of construction is the only obstacle preventing these admirable sheaves from coming into more general use.

Fig. 77.
White's Pulley.

Since the fall passes over all the sheaves in succession, each part having the same tension, and the tension of the first part being equal to the power, the weight is as many times the power as the number of parts of the fall rove in the movable block is to the hauling part. Thus, in the figure of the luff-tackle (fig. 68), the weight is to the power as 1 to 4, since there are 4 parts of the rope on the movable block.

Effect of Angles made by the Fall.—It has been shown that the effect of a force applied to a rope passing over a fixed pulley is the same, whatever the angle of the two parts of the fall may be. When the fall is rove through a *movable* block, and one end is fixed, the greater the angle between the two parts of the fall, the less will be the lifting force.

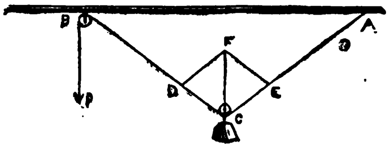

Fig. 78.

Let the weight C (fig. 78), be supported by the rope A B, passing through the pulley C, the directions of whose parts, A C, B C, are not parallel. It is evident that C is acted upon by two equal forces, A C and B C.

Take any distance, C D, in C B, to represent the force C B, and cut off C E on C A equal to C D. Complete the parallelogram C D F E, and join F C. By the parallelogram of forces the diagonal E C is the resultant of the two forces of C B and C A, whose length will represent the effective force of the two forces C B and C A. Now, if C D equals two units and F C three, it is evident that one unit of force is lost, as C D and C E equal four units. The diagonal F C representing the efficiency of the two forces C A and C B, each of which is equal to the power, half the diagonal is equal to the power. Hence, *as the difference between half the length of the diagonal and the side of the parallelogram becomes greater, so does the difference between the efficiency of the power and the power itself increase;* or,—

A power equal to that represented by two sides of the parallelogrom will have to be exerted to produce an effect equal to that represented by the diagonal. Wherefore,—

The greater the angle between the two parts of the fall, the greater is the loss of efficiency in the machine.

Virtual Velocities.—The principle of virtual velocities is applicable to all systems of pulleys, for the weight ascends through a space as many times smaller than that through which the power descends in the same time, as the weight is greater than the power; the moments of the power and weight are therefore equal.

Rules for ascertaining the mechanical efficiency of pulley systems.

First. With one cord.—Divide W by the parts of the fall in the movable block.

Example.—To raise 600 lbs. with a system having 6 sheaves in the lower block, and the end fastened to the upper block:—

$$\frac{600}{6 \times 2} = 50 \text{ lbs.}$$

When the end is fastened to the lower block:—

$$\frac{600}{6 \times 2 + 1} = 46{\cdot}15 \text{ lbs.}$$

In other words, $W = n \times P$, n being the number of parts of the fall in the movable block.

Second. With more than one cord.—When the ends are fastened to the support. $W = 2^n \times P$.

Example.—What weight will 1 lb. support, with 4 movable pulleys and 4 ropes? $1 \times 2 \times 2 \times 2 \times 2 = 16$ lbs.

When fixed pulleys are used to fasten the ends to the support. $W = 3^n \times P$.

Example.—What weight will be supported by 5 lbs. and 4 movable and 4 fixed pulleys and 4 ropes? $5 \times 3 \times 3 \times 3 \times 3 = 405$ lbs.

When the ends of the rope or fixed pulleys are fixed to the weight. $W = (2^n - 1) \times P$, and $W = (3^n - 1) \times P$; or, $1 \times 2 \times 2 \times 2 \times 2 = 16 - 1 = 15$, and $5 \times 3 \times 3 \times 3 \times 3 = 405 - 1 = 404$.

The principle of virtual velocities, as applied to the pulley, is proved as follows. Suppose the lower block has four sheaves, and the parts of the fall are parallel. If the weight be raised through a height X, the whole of the eight parts of the fall must be shortened by X, and the power must descend through 8 X. Wherefore $\frac{X}{H} = \frac{X}{8X} = \frac{1}{8}$ H, where H equals the descent of the power.

CHAPTER X.

THE MECHANICAL POWERS (*continued*).

4. *THE INCLINED PLANE.*

Description.—The inclined plane is a sloping surface, supposed to be perfectly inflexible.

It may be compared to the hypothenuse of a right-angled triangle (fig. 79), whose height is small com-

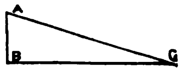

Fig. 79.

pared with the length of its base. The inclined plane is used to move heavy bodies horizontally and vertically over moderate distances, when the force applied is small.

A familiar example of the inclined plane is seen in the contrivance used by draymen to raise heavy cases and casks into their drays. The cask, requiring greater power to raise it than can be directly supplied by the strength of the men, is pushed up a sloping surface. The steeper the slope, the greater the power required to raise the body. Hence, the angle of the inclined plane is usually small.

Definition.—The inclined plane may be defined to be a machine consisting of a sloping surface, by means of which a small force applied over a greater distance balances a weight as many times greater than the power as the perpendicular distance through which the weight moves is less than the horizontal distance over which the power moves.

Conditions of Equilibrium.—A body at rest upon a horizontal surface is in equilibrium under the influence of the two equal and opposite forces of gravity and reaction.

But when the surface upon which the body is placed is not horizontal, a third force is required to produce equilibrium.

This is the case with bodies upon an inclined plane; and the third force may have a direction making an angle with the plane, parallel to the plane, or parallel to the base. Into one of these three sections every problem connected with this branch of mechanics must fall; and we will proceed to consider them seriatim.

First. The Power acting at an angle with the Plane.—Let A (fig. 80) be a body resting on the inclined plane B C, and supported by a power in any direction, as A P. A will remain at rest by the effect of three forces acting upon its centre of gravity, A.

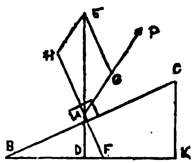

Fig. 80.

1st, *Gravity*, acting along the vertical, A D;

2nd, the *Power*, P, acting in the direction A P; and,

3rd, the *Reaction* of the plane B C, acting perpendicularly to it, that is, in the direction F A.

Now, since the force A D is opposed and equal to the two forces, F A, A P, it must be equal to the resultant of these two forces. Suppose the line D A be extended upwards in a straight line to E, so that A E equals the force A D. From the point E, draw E G and E H, respectively parallel to A H, or F A produced, and A P.

Since A E is equal to the weight, A G must be equal to the power, and A H to the reaction of B C.

This may be proved by experiment. Let a cord be fastened to A, and pass over a pulley in the direction of C, and a weight be attached to it bearing the same proportion to the weight of A that the line A G does to the body A E. At the same time, let a cord be passed over a pulley in the direction A H, to which a weight proportionate to A H is suspended. Upon the removal of the inclined plane B C, the weight A will be found

to remain suspended in its original position. Hence, the weight H supplies the place of the plane, B C, and produces the same mechanical effect.

If two triangles be drawn, having the angles of the one equal to the angles of the other, each to each, the respective sides of the two triangles are proportional. Hence, if a triangle be drawn, whose angles are equal to those of a triangle, one side of which is vertical, a second perpendicular to the plane, and the third in the direction of the power, *the sides of the triangle so constructed will be proportioned to the power, the weight, and the pressure on the plane.*

Therefore, in fig. 80, since in the triangle A H E, A E is vertical, A H perpendicular to the plane, and H E in the direction of the power, the side A E equals the weight, A H equals the pressure on the plane, and H E the power.

Second. The Power acting parallel to the Plane.—Let A B C D (fig. 81) be a weight resting upon the in-

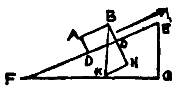

Fig. 81.

clined plane E F G, of which E F is the length, E G the height, and F G the base. Take any length to represent the weight of A B C D, and from B draw B K in the direction of the vertical, equal to that length. From K draw K H in the direction of the power, P, and produce B C in the direction of the pressure, to meet K H in H.

In this case, the base, F G, of the inclined plane represents the pressure; the height, G E, the power; and the length, E F, the weight. And the triangle B K H, having been formed by lines representing the power, the weight, and the pressure, the two triangles are similar.

Wherefore, *when the power is in the direction of the*

plane, the power is to the weight as the height of the plane is to its length.

In other words, the force must equal a sum found by multiplying the weight by the rise of the plane in a given length, and dividing by the length. For example, if the rise be 3 feet in 10 feet, the weight will be supported by a power equal to $\frac{3}{10}$ of the weight.

Third. The Power acting parallel to the Base.—Let W (fig. 82) be the weight pressing on the plane in

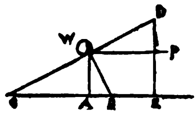

Fig. 82.

the vertical W A, and let P be the direction of the power. The resistance of W will be W B, perpendicular to the plane.

Let W A equal the weight. From A draw A B, parallel to W P, to meet W B in B. A B is proportioned to the power in the same ratio as W A is to the weight, The triangle W A B being equiangular to the triangle C D E, is similar to it; and, therefore, the weight is represented by the base, the pressure by the length, and the power by the height of the triangle C D E.

Therefore, *when the force exerted on a body resting on an inclined plane is parallel to the base of that plane, the power is to the weight as the height of the plane is to the length of the base.*

It is evident, therefore, that for the power to act with the greatest advantage, it must move in a direction parallel to the plane.

Double Inclined Plane.—When two weights rest on two inclined planes, and are joined by a cord passing over a pulley at the junction of the planes, the proportion of the weight of the one to the weight of the other necessary to establish equilibrium will be as the lengths of the planes upon which they rest.

Thus (fig. 83), let A and C be two weights connected

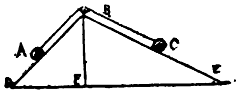

Fig. 83.

by a cord passing over a pulley, B. The proportion that A bears to C will be as the plane D B is to B E.

Suppose B F to represent the power, B D the weight, and D F the pressure in the one plane; and B F to represent the power, B E the weight, and E F the pressure in the other plane. The power, B F, being common to the two systems, it follows that the weights of A and C are as the lengths D B and E B, and that the pressures of A and C are as D F to E F.

Application.—The uses to which the inclined plane is applied are very numerous. One of the commonest examples is with a brewer's dray, where the barrel is rolled up an inclined plane, A B (fig. 84).

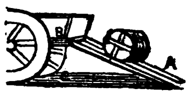

Fig. 84.

In this instance, the weight is lifted over the distance B C by exerting a power bearing the same proportion to it that B C bears to A C over the distance A B.

Rule for determining the mechanical efficiency of the inclined plane. As the length is to the height, so is W to P.

Example.—What P will raise 1000 lbs. up an inclined plane 6 feet long and 4 feet high? 6 : 4 :: 1000 : 666·66.

Let W=weight, h=height, l=length, P=power, b=base, and p=pressure.

$$\frac{W \times h}{l} = P; \quad \frac{P \times l}{h} = W; \quad \frac{W \times b}{l} = p.$$

To find the length of the base, height, or length of a plane, when he other two are given. For the length of base. Subtract the square of h from the square of l and $\sqrt{\text{remainder}} = b$.

To find the length of the plane. Add the squares of the other two dimensions, and $\sqrt{\text{sum}} = l$.

To find the height. Subtract the square of b from the square of l, and $\sqrt{\text{remainder}} = h$.

Example.—The height of a plane is 20 feet, the length 100 feet. What is the length of the base, and what pressure will 1000 lbs. exert on the plane?

$$\sqrt{100^2 - 20^2} = \sqrt{9600} = 97 \cdot 98 = \text{base.}$$

100 : 20 :: 1000 : 200 lbs. = force necessary to raise the weight; and

$$\frac{1000 \times 97 \cdot 98}{100} = 979 \cdot 8 \text{ lbs. pressure.}$$

CHAPTER XI.

THE MECHANICAL POWERS (*continued*).

5. *THE WEDGE.*

Definition.—A wedge is a machine consisting of a triangular prism[1] (fig. 85) composed of two inclined planes, joined at their bases.

It is used to exert great pressure over a small distance where but little motion is required. The difference in the mode of applying a wedge and an inclined plane is, that in the latter the weight is drawn along the surface of the plane by a continuous force, while in the former the plane is made, by a blow, to move between the surfaces to be separated, or beneath the body to be raised. The wedge may, therefore, be called a *movable inclined plane.*

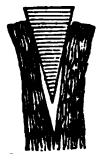

Fig. 85.

The resistances act perpendicularly to the sides of a wedge, and the force is applied perpendicularly to its base.

Pressure and Percussion.—In other machines, the force employed is a *continuous* effort, called *pressure;* but

[1] Gr., *prisma*, sawn out. A solid body whose ends are similar, equal, and parallel plane, and whose sides are parallelograms.

in the case of the wedge the power is a *momentary* effort, called *percussion*.[1] So different are these in their effects, that a pressure of many tons may not have a resultant effect equal to that of a percussion of a few pounds.

It has been thought that no resistance is so great that it cannot be eventually overcome by percussion, however slight. The axiom, therefore, to be deduced from such an hypothesis is, that *a blow, however slight, if continually repeated, is equal to a pressure, however great.*

As the wedge is only used by percussion, and as the theory of the inclined plane does not apply to such forces as impacts,[2] the consideration of the true theory of the wedge becomes a matter of some difficulty, inasmuch as the weight to be overcome is a *pressure*, and the power by which it is to be overcome is a *blow*. One fact is evident, that the smaller the ratio between the base and the sides of the plane, the greater is the efficiency of the machine. This rule holds good with respect to the inclined plane itself.

The vibration of the blow may cause a momentary separation of the surfaces, and the wedge, impelled by the blow, holds the surfaces apart.

Friction.—In every other machine, the friction is a great drawback. In the wedge, however, this friction is the greatest aid. Indeed, without friction, the reaction would cause the wedge to recede as far as it had been advanced by the preceding blow.

The friction of the two faces being sufficient to prevent the recoil, *no other force is necessary to keep the machine in equilibrium.*

Application.—The uses to which the wedge is applied are, to raise heavy weights to a small height, or to cleave timber, a small notch having been previously cut to receive the point of the wedge.

As an instance of its great power, when a ship is

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[1] L., *percussio*, to strike or shake thoroughly. [2] L., *in*, against; and *pango*, to strike.

built on stocks and is ready for launching, her cradle is built under her, and then her weight is transferred from the stocks to the cradle, the ship being lifted off the stocks by the simultaneous action of a number of wedges.

All cutting instruments, such as knives, razors, chisels, etc., act as wedges. The harder the substance to be cut, the more obtuse is the angle of the wedge required to be.

RULES.—To determine the force necessary to force two bodies apart when both the bodies are movable.

As the length of the wedge : ½ the back :: resistance : P.

Ex. 1. The length of the back of a double wedge is 6 inches, and its middle length is 10 inches. What force will separate a resistance of 150lbs? 10 : 3 :: 150 : 45lbs.

When one of the bodies is fixed.

As the length of the wedge : back :: resistance : P.

Ex. 2. The breadth of a wedge is 6 inches, and the length 100. What force will raise 15,000lbs? 100 : 6 :: 15,000 : 900lbs.

In these rules friction is not taken into consideration.

CHAPTER XII.

THE MECHANICAL POWERS (*continued*).

6. *THE SCREW.*

Definitions.—This powerful machine is generally employed to exert great pressure. It may be described

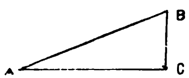

Fig. 86.

as a helix,[1] that is, an inclined plane revolving round a

[1] L., *helix*, a snail. From the spiral shape of the shell.

centre. If a piece of paper be cut to the form of an inclined plane, as A B C (fig. 86), and wrapped round a cylinder, the length of the plane will mark a spiral line round the cylinder, which line is called the *thread of the screw.* The distance between the coils of the thread is

Fig. 87.

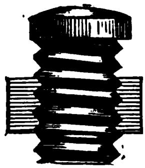

Fig. 88.

called the *pitch of the screw.* The threads projecting from the cylinder, are either square (fig. 87) or triangular (fig. 88).

The screw is usually connected with a hollow cylinder, the inside of which is cut with a spiral groove corresponding to the projections on the screw. This is often called the *nut;* and the screw and nut are called respectively *male* and *female screws.*

Nature of Screw.—If a mark be placed on one of the threads, and the screw be turned round once, the mark will be found to have advanced the distance between two threads, because the screw cannot revolve without sliding along the inclined plane of the threads.

A screw may, therefore, be compared to a triangle having the height, A B (fig. 89), equal to the pitch, the

Fig. 89.

base, B C, equal to the circumference of the cylinder, and the hypothenuse, A C, equal to the inclined plane. The

power, therefore, will be to the weight as A B is to the base, B C; or,

The power is to the weight as the pitch is to the circumference of the cylinder.

There appears to be, therefore, no limit to the proportion between the power and the weight. But if we decrease the pitch, we must also decrease the size of the threads; and, consequently, they may ultimately become too weak to withstand the pressure of the weight and power.

This was the case in the wheel and axle; but the defect was remedied by having axles of different diameters, round which the rope was wound in the same direction.

Differential Screw.—The application of this principle to the screw by Dr. Hunter, enables us to alter the proportion between the pitch and diameter to any degree.

The board B (fig. 90) moves in a groove between the

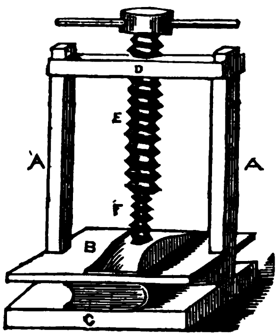

Fig. 90.

uprights A A; consequently, any substance placed between it and the bottom, C, will be subjected to any pressure placed upon the board. D is another board

fixed to the frame and having a female screw cut through it, to allow the screw E to work. E is a male screw working through D, and having a female screw cut inside, adapted to receive the screw F. When the screw E is turned once round, it will advance through D the distance of its pitch; and if it were connected with B, B would advance the same distance nearer the bottom C. But the motion of E causes the screw F to move towards it by the distance of the pitch of the screw F; consequently the board B will move upwards through the same space as the pitch of F. If, now, the pitch of F equals the pitch of E, the tendency of E to lower B will be exactly counteracted by the tendency of F to raise B. But if the pitch of F be a little smaller than the pitch of E, B will move downwards through a space equal to the difference between the pitch of E and F. Thus, if the screw E have a pitch of $\frac{1}{20}$ of an inch, and F have a pitch of $\frac{1}{21}$ of an inch, D will move downwards in one revolution one twentieth of an inch and upwards one twenty-first of an inch, the effect of which will be that D will descend the difference between $\frac{1}{20}$ and $\frac{1}{21}$ of an inch, or $\frac{1}{420}$ of an inch. The effect of this arrangement, therefore, is the same as if the screw had 420 threads to an inch, with the advantage of having the threads strong enough to resist any pressure that may be put upon them.

Application of the Power.—We have hitherto considered the power to be applied to the circnmference of the screw; but in practice this is never done. The screw is in fact a *compound* machine; for the power is applied to a lever passing through the head of the screw, the same as the power is applied to the wheel and axle. The efficiency of the machine is thereby greatly increased. for we not only gain the effect of the screw itself, but also that of the lever.

In estimating the power of a screw, the length of the lever at the end of which the power is applied, must be taken into consideration. The formula for obtaining

this is as follows:—*The power, multiplied by the circumference it describes, is equal to the weight multiplied by the pitch of the screw.*

In some machines the lever is attached to the female screw or nut, and the screw is fixed, or the nut may be capable only of a revolving motion and the screw only of a longitudinal one; but the principle in all cases is the same.

Micrometer Screw.—As a screw admits of a very small progressive motion being imparted to it by means of a great motion of the power, this fact is taken advantage of in the measurement of very small spaces, by what is called a *micrometer*[1] *screw.* Thus, suppose a screw have a pitch of one fortieth of an inch; in one revolution it will advance one fortieth of an inch. But if the head of the screw consists of a large disc, we can easily divide one revolution into 100 parts. Hence, if we move the screw over $\frac{1}{100}$th of a revolution, we advance it $\frac{1}{40}$th of $\frac{1}{100}$, or $\frac{1}{4000}$th of an inch.

This principle is applied in the *tangent*[2] screws of sextants and other scientific instruments, where a large screw moves the index over the larger division of the scale and a small micrometer screw moves it over the small divisions of each of the larger ones.

The Endless Screw.—The male screw, instead of acting upon a female screw or nut, is sometimes connected with a toothed wheel. This arrangement is called an *endless screw* (fig. 91), since the teeth of the wheel will continually succeed each other as they advance, so that they never arrive at the end of the screw.

It is evident that this is a machine compounded of the wheel and axle and the screw. The power acting upon the teeth of the wheel is the increase of power applied to A, gained by means of the winch A and the

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[1] Gr., *micros*, small; *metron*, a measure. [2] L., *tango*, to touch. A tangent being a line touching a circle.

screw B. The machine may therefore be considered to be acted upon by two powers; the first applied to A, and the second acting at C on the teeth of the wheel.

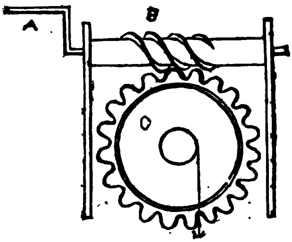

Fig. 91.

This machine is very powerful; but its action is very slow, thereby affording another example of the truth of the axiom, that what is gained in power is lost in speed.

The formula for calculating the force exerted by a screw, and that by which the mechanical advantage of a wheel and axle is ascertained, have been given above. By compounding these two formulæ we obtain the following equation:—*The power is to the weight as the pitch multiplied by the radius of the axle is to the circumference described by the power multiplied by the radius of the wheel.*

For example, If a power of one pound be applied to the rim of a wheel whose circumference is thirty-six inches, the pitch of the screw be one inch, the radius of the wheel be twelve inches, and the radius of the axle be four inches, the weight is to the power as fifty-four to one.

To estimate the efficiency of a screw. The length of the inclined plane composing the screw = the square root of the sum of the squares of the circumference and of the pitch, and the height is the distance between the consecutive threads.

Rule.—As l of the plane is to pitch, so is W to P. P = power,

R = length of lever, W = weight, l = length, p = pitch, x = effect of power at the circumference of the screw, and r = radius. Then

1. $l : p :: W : P$.	5. $P : W :: p : l$.
2. $l : W :: p : P$.	6. $r : R :: P : x$.
3. $W : l :: P : p$.	7. $P : x :: r : R$.
4. $p : l :: P : W$.	8. $R : r :: x : P$.

Example.—What power will raise 8000 lbs. by a screw 12 inches in circumference and 1 inch in pitch?

$$\sqrt{12^2 + 1^2} = \sqrt{145} = 12{\cdot}04159 = l.$$
$$\text{Then, } 12{\cdot}0416 : 1 :: 8000 : 664{\cdot}36 \text{ lbs.} = P.$$

COMPOUND MACHINES.

We have already had an example of a compound machine in the case of a screw in which the power is applied through the medium of a lever, and in the endless screw. This, however, being a combination always understood when speaking of a screw, is never considered as a compound machine.

The mechanical powers admit of endless combinations, so that a given power may be made to exert a given force in any required direction.

A combination of levers is used in the ordinary road weighing machine.

Combinations of wheels and axles may be witnessed in the crab winch and the clock. Indeed, most machines are combinations of wheels, axles, and levers.

The screw is also continually used combined with other mechanical powers.

We have an instance of the inclined plane in a door latch, and, in combination with the wheel and axle, in the machinery used to draw waggons up the slope of a hill to a mine situated on the top.

In all machines, however, the principle holds good, that *what is gained in power is lost in time*, and the converse. For if we can, through the medium of machinery, by the exertion of one pound, lift a weight of ten pounds, we should, by the application of ten pounds of direct force, lift the same weight in one tenth the time without the intervention of machinery.

CHAPTER XIII.

MEANS OF CONVERTING MOTION.

Divisions.—It very rarely happens that a mechanical agent acts in such a manner as to admit of its being applied direct to the working point. In the steam-engine, for instance, the power is applied to drive a piston alternately from one end of a cylinder to the other; and this reciprocating[1] rectilinear[2] motion has to be converted into a continuous circular motion. In short, in every machine we must modify the motion of the power in such a way as to produce the required motion in the weight.

The principal modifications of motion[3] are as follows:—

1. Converting reciprocating rectilinear motion into continuous circular motion, and *vice versâ*.
2. Converting reciprocating rectilinear motion into reciprocating circular motion, and *vice versâ*.
3. Converting continuous circular motion in one plane into continuous circular motion in another plane.
4. Converting continuous circular motion into continuous rectilinear motion.

1. Converting reciprocating rectilinear motion into continuous circular motion, and vice versâ.—This is the most common modification of motion.

Crank.—The simplest contrivance for effecting it is the *crank*,[4] two drawings of which are annexed. It consists of a winch or elbow, A (figs. 92 and 93), rigidly fixed to the axis of the wheel to which con-

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1 L., *reciproco*, to act interchangeably. 2 L., *rectus*, right; *linea*, a line. 3 Motion is said to be reciprocating when it moves forward to a certain point and then returns to its original position, and so on. When the path is a straight line, the motion is said to be *reciprocating rectilinear*, and when circular, *reciprocating circular motion.* 4 Dutch, *kring*, a circle.

tinuous circular motion is required to be given. To the crank a long rod, called a *connecting-rod*, B, is attached

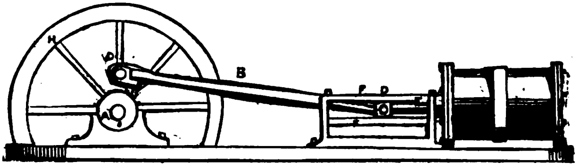

Fig. 92.

by a joint at C, and traverses the circumference of a circle of which the *throw* of the crank, A C, is the radius. The other end, D, is hinged to the *piston-rod*, D E, by a joint called the *cross-head*, so that when the piston-rod moves alternately backwards and forwards between the guide-bars F F, one end, D, of the connecting-rod moves with a reciprocating rectilinear motion and the other end, C, with a continuous circular motion.

Fig. 93.

The greatest efficiency of the connecting-rod in turning the axle, is obtained when the crank and connecting-rod are at right angles to each other. In every other position a portion of the power is spent in pulling the crank away from the axle. When the piston-rod, connecting-rod, and crank are in a straight line with each other—which is the case twice in every revolution—the power has no tendency whatever to turn the axle. These are called the *dead points* of a machine; and the crank is carried past them by the momentum of the heavy parts of the machinery. Still the motion of the axis would be considerably retarded at the dead points,

and accelerated when the crank was in the position of greatest efficiency, were it not for a simple method of equalizing the speed. This contrivance, called a *fly-wheel*, consists of a large wheel with a heavy rim, fixed upon the axis of the crank. The fly-wheel receives momentum when the connecting-rod acts with greatest advantage, and expends it as the advantage decreases, so as to carry it past the dead points. The crank is therefore carried round continuously with an approximately uniform motion.

Excentric.—It is evident that by the same contrivance a circular motion can be converted into an alternate rectilinear motion; but the most general way of effecting this is by means of the *excentric.*[1] The connecting-rod, A (fig. 94), is fixed to a metallic ring, B B, the inner

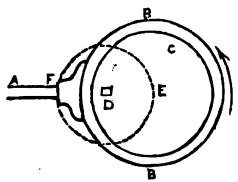

Fig. 94.

surface of which is perfectly smooth. C is a metallic plate fixed to the axle of the fly-wheel at a point, D, between the centre of the plate E and the circumference. The ring B fits a groove cut in the rim of the plate, so that C can turn freely within it. The centre, E, of the plate will describe a circle round the centre, D, of the shaft, the radius of which will be the distance E D

The disc and shaft form an excentric.

While the plate revolves, it will be alternately driven from one side to the other in the revolution, since the ring does not partake of its revolution. In the figure,

[1] L., *ex*, out of; *centrum*, the centre.

the point E is to the right of the shaft; if, now, the excentric revolve in the direction of the arrow, the point E will ascend, carrying the band B with it; and when half a revolution has been accomplished the point E will have arrived at F. The distance through which the point E moves is called the *throw* of the excentric, and is equal to twice the distance between the centres of the shaft and disc. The connecting rod, A, has therefore the same motion as if attached to a crank having the length of D E.

2. **Converting reciprocating rectilinear motion into reciprocating circular motion, and vice versa.**

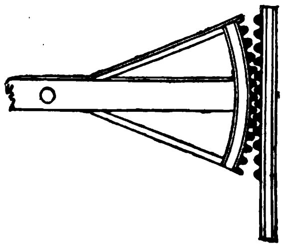

Fig. 95.

Rack Motion.—The piston-rods of Bolton and Watt's engines were furnished with a rack on the upper end, which engaged with a toothed sector at the end of the beam, forming a tangent to it (fig. 95). By this method, the reciprocating rectilinear motion of the piston-rod was modified into the reciprocating circular motion of the beam.

Parallel Motion.—This, however, was found to work very unsteadily; and at length Watt invented his parallel motion, which attained the desired end.

Let A (fig. 96) be the centre of the beam. Let its position at its greatest elevation be A B, and its lowest A C; the end will describe the curve B C. If the piston-rod were attached to B, it would be strained al

ternately to the right and to the left at each stroke; and

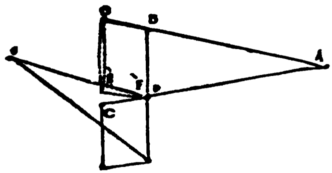

Fig. 96.

the object is to preserve the perfect perpendicularity of the piston-rod. Let any point, D, be taken on the beam near to the extremity C. Two rods, C E, D F, of equal length are attached to C and D, and their extremities are joined by a third rod, F E, equal in length to D C. C D F E is therefore a parallelogram. The piston-rod is connected at E. At F another rod, F G, is joined, hinged at G to a fixed point in the plane of the parallelogram, but outside it. Since the points D and C play in arcs whose centre is A, their effect is to throw the piston-rod to the right. The joint F G, or *link*, moves on the fixed centre G, and plays in an arc whose effect is to throw the piston rod to the left. The two arcs, therefore, act contrary to each other; and the proportion of the lengths of the rods is so adjusted that the effect of the rod G F in throwing the piston rod to the left, exactly counteracts the effect of the beam in throwing it to the right; consequently the end of the piston-rod moves up and down in a straight line.

3. **Converting continuous circular motion in one plane into continuous circular motion in another plane.**

Bands.—Where the resistance is small, and the drums are not required to be very near each other, this change of motion may be accomplished by a strap passing over drums, as in fig 97, where the strap A A, passing over the drum B, drives it in a vertical position, then passing

over the drum C, drives it in a horizontal position,

Fig. 97.

and finally passing over the drum D, drives it in a horizontal position at right angles to C.

Bevelled Wheels.—But where the resistance is very great, rotation may be transmitted by means of bevelled[1] wheels. These are toothed wheels fixed on conical webs, as in fig. 98, where the wheel A engages with the wheel B and drives it at right angles to A.

Fig. 98.

When the teeth are cut parallel, perpendicular to the

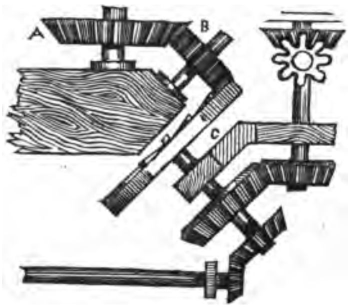

Fig. 99.

axis and in the direction of the radii, the wheel is called a *spur wheel.* The wheels on page 66 are spur wheels.

When, however, the teeth are parallel to the axis of the wheel, as in C, fig. 99, the wheel is called a *crown wheel.*

The angle of the shafts of two bevel wheels working

[1] F., *biveair*, a slant; or Sp., *bay-vel.*

together may be varied at pleasure by altering the angle of the cone of the web. Fig. 99 shows a bevel wheel, A, with a small angle, driving a spur wheel, B, which drives a crown wheel, C.

4. **Converting Continuous Circular Motion into Continuous Rectilinear Motion.**—The wheel and axle is an instance of this, in which the continuous circular motion of the axle causes a continuous rectilinear motion of the rope.

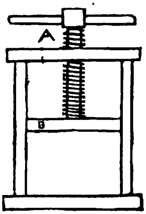

Fig. 100.

If a screw, A (fig. 100), have only a motion round its axis, and the plate B have a female screw adapted to receive A, the circular motion of A will impart to B a rectilinear motion.

Many other modifications of motion are possible; but a description of the mechanism by which they are produced would occupy more space than can be devoted to the subject in this work.

CHAPTER XIV.

FRICTION.

Omitted Resistances.—In estimating the mechanical efficiency of machines, we have hitherto supposed the power to be communicated to the weight without the intervention of any of the detracting influences which act upon all bodies, whether in motion or at rest.

We have supposed that the surfaces of all bodies were perfectly smooth; that axles and fulcra were mathematical lines and points; that surfaces moving in contact were absolutely without friction; and that ropes were perfectly flexible, and possessed neither weight nor thickness.

These suppositions, it must be evident, can never obtain in the practical application of the mechanical

powers. Surfaces, however polished, still possess sufficient roughness to create friction[1]; axles and fulcra are always cylinders and truncated[2] cones; and ropes require considerable force to bend them round their respective pulleys, and possess, in addition to a very appreciable thickness, considerable weight.

Various means have been adopted to smooth down the asperities between two surfaces in motion, by the intervention of oil or other unctuous matter, or by mechanical arrangements. But in spite of all our endeavours, the power applied to any machine is considerably diminished before it reaches the working point. Hence, machines, so far from augmenting force, in reality *detract from it.* That is to say, the application of the force to the weight to produce a given motion, could be more expeditiously and economically effected by a *direct* application to the weight, than by the intervention of machinery.

So great is the detracting power of friction, that at least one-third, and often one-half of the entire moving force employed, is consumed in overcoming it.[3]

Active and Passive Forces.—Force may be considered to be of two kinds, distinguished from each other by their effects. The one kind is capable not only of diminishing motion, but also of transmitting and accelerating it. Such force is called *active.*

The other kind is capable of diminishing motion, but incapable of transmitting or accelerating it. Such force is called *passive.*[4]

No active force can be called into existence without creating passive force; and according as the proportion between the active and passive forces is great or small, so is the mechanical efficiency of the active force more or less diminished.

The resistance offered by a fluid or solid to a body

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[1] L., *frico*, to rub. [2] Cut off. [3] See volume on Heat in this series. [4] L., *passus*, borne, suffered; as opposed to active.

moving over or through it, constitutes a passive force. It is in consequence of the effect of passive forces that it is necessary to make active force continual in order to keep up a continual motion.

Passive forces have a different effect when applied to machines in motion and to machines in equilibrium. When the machine is in equilibrium, the passive force assists the power; that is to say, less power is necessary to support the weight than if the passive force did not exist.

For instance, theoretically, a weight of two pounds resting on an inclined plane whose length is twice its height, requires a force of one pound to sustain it in equilibrium. But practically no such force is necessary; for the passive force, or friction, between the surfaces of the weight and plane is so great as absolutely to require a force to overcome it. The very opposite effect is experienced in the case of bodies in motion; for to insure the motion of a body, a power equal to the resisting force must be added to that required to overcome simply the weight of the body.

Thus, in the case of a whip over a single pulley, the smallest possible force applied to the hauling part in excess of the equivalent of the weight, should move the weight. But the stiffness of the rope, and the friction of the sheave against the shell and pin, are such resisting forces, that a considerably greater force than the equivalent of the weight is necessary to move it.

In the case of the wedge, the resisting force is alternately a disadvantage and an advantage. In splitting wood, for instance, the friction between the faces of the wedge and the timber opposes considerable resistance to the advance of the wedge from the effect of the blow. But when the advance has ceased, the friction becomes advantageous, for it prevents the wedge from recoiling. Were there no friction, we should be obliged to drive the wedge by continuous pressure, instead of by the much more convenient force of percussion.

More power is gained by the friction between the strokes of the hammer, than is lost by its resistance to the advance of the wedge.

Of all resisting forces, that of friction is by far the most important.

It may be divided into two classes. First, *sliding friction*, or friction of one surface sliding upon another; and, second, *rolling friction*.

I. **Sliding Friction.**—No surfaces are absolutely smooth; otherwise there would be no friction. Even the most highly polished surface, when viewed with a magnifying glass, exhibits roughness. When a body is placed on a surface, each of the faces in contact possessing inequalities, elevations of one lock into depressions of the other. When a sliding motion is communicated to one or both of the bodies, the body must be carried up the inclined planes of the surfaces of the asperities. This is illustrated by fig. 101, in which A and B are two bodies, whose surfaces, CD and EF, in contact, have their roughness exaggerated for the sake of illus-

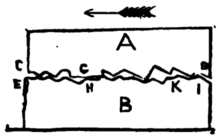

Fig. 101.

tration. Now, to move A in the direction of the arrow, it is evident that unless the force is sufficient to raise it over the inequalities of B, it will not move. Thus, the point D must be lifted over the point K, and so on. When the rectangular mass G arrives at H, it must either be lifted over H, or G must be broken off. In practice the asperities *are* broken off, and the loss of the substance causes the wear of the surfaces in contact.

Now, as A must be lifted over the elevations of B, it follows that *the greater the weight of A, the greater must be the force necessary to move it;* and that, *however much*

the surface of the faces in contact be enlarged or reduced, provided the weight of the body remains the same, the friction or force required to move the body is the same.

This may be proved by experiment. Let A B (fig. 102) be a table, and C a piece of wood having a cord, D,

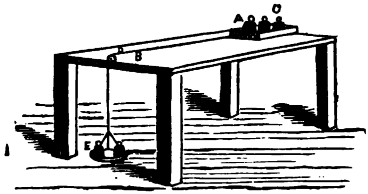

Fig. 102.

to which a scale-pan, E, is suspended, passing over a pulley. Suppose C to weigh two pounds. If E be filled with sand until its own weight and that of the contained sand equals two pounds, C and E will be in equilibrium. Now let sand be poured into E until its weight is just sufficient to move C. The weight of the additional sand represents the friction.

If now C be loaded with two more pounds, and sand be poured into E until C again moves, it will be found that the quantity poured in under the double pressure is double that previously required to move C. The friction therefore increases in the same ratio as the weight, or pressure. There are a few exceptions to this rule, in which the friction increases in a slightly less ratio than the pressure.

Now let C be turned on its edge, and suppose the surface of its edge to be only one half the surface of its face. When sand is poured into E, it will be found that it requires as great a weight to move it as when C was on its face.

If the weight of C be two pounds, and the surface of the face of C be sixteen square inches, the pressure on each square inch will be two ounces. And if the edge

of C be eight inches, the pressure on each square inch will be four ounces; the total pressure in each case being thirty-two ounces, or two pounds.

Wherefore, *the friction is proportional to the pressure, irrespective of the area of the surfaces in contact.* By this experiment we obtain the proportion of the friction to the pressure between the surfaces of the bodies experimented upon; viz., *As the weight of the excess of sand in E necessary to produce equilibrium is to the weight of C, so is the friction of C to the pressure of C.*

Another way of demonstrating this ratio, is by means of the inclined plane. Thus, in fig. 103, let A C be a plane capable of being placed at any angle by the screw B C, and upon it let a weight, W, be placed, the ratio between the friction and pressure of which it is desirable to ascertain. Let the screw B be turned until A C attains such an angle as will just cause the weight W to move down the inclined plane. The force down the plane will then represent the friction. If A C equals the weight, B C will equal the force down the plane, and A B the pressure.

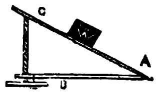

Fig. 103.

The friction is to the pressure, therefore, as B C to A B. If the weight be increased, the same elevation will cause it to move. The angle is called *the angle of repose*, or limiting angle of resistance, and the ratio between the force tending to make the body slide and the force pressing it against the plane, namely B C and A B, is called the *co-efficient of friction.*

Axioms.—From these experiments the truth of the following axioms will be evident.

1. *When the surfaces are the same, the friction is proportional to the pressure.*

2. *Friction is independent of the area of the surfaces in contact.*

3. *Friction is independent of the velocity of the body;* hence—

4. *Friction is a uniformly retarding force.*[1]

Although friction is a deteriorating force to bodies in motion, yet when a body is required to be kept at rest, it is an advantage. Without it the wedge would be a useless machine, for it would recoil after each blow; and nails driven into a board would not hold. By friction the ropes of a ship are fastened by merely twisting them round a belaying-pin. Indeed, without friction, the aspect of things in general would be very much changed; horses could not draw loads, nor could men or animals walk, and they would helplessly slide to the bottom of any slope upon which they were placed.

II. **Rolling Friction.**—When a body rolls over the surface of another, the particles, instead of being drawn on over each other, are merely laid down and lifted up. As the surface of one of the bodies, at least, must be bounded by a curved line, which can only touch another line at a line or a single point, the surfaces of contact are reduced to a minimum.

If the rolling body be a cylinder, the points of contact will be a line parallel to the axis of the cylinder; but if the body be a globe, the area of contact will be a point.

Cylinders.—To ascertain the ratio between the friction and pressure of rolling bodies, we will adopt the angle of the inclined plane as our test. Thus, in fig. 104, let A B be the inclined plane, upon which the

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[1] That is to say, that the amount of force expended in overcoming friction is proportional to the velocity, since a constant force must be exerted with a varying velocity.

Hence, whatever be the ratio between the force and friction of two moving surfaces of any machine moving with a given velocity, when the velocity is doubled,—the pressure on the surfaces remaining the same,—the ratio between the force and friction will also be doubled.

It also follows, that the amount of force lost by friction will in every case be proportional to the space passed over by the surfaces in contact, without regard to time.

Friction, therefore, is always the same fraction of the pressure, however great (within certain limits), and however small that pressure may be.

cylinder H is placed, of the same weight as the body used in the former experiments. By the elevation of

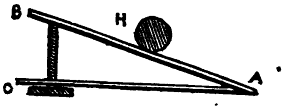

Fig. 104.

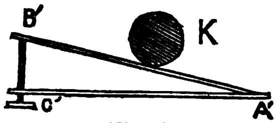

Fig. 105.

the screw, the angle of repose is found to be much smaller for equal weights when the surface of one of the bodies is round, than where both the surfaces in contact áre flat. In this case, suppose H to be of twelve pounds weight. The friction is to the pressure as B C to A C.

Again; let K (fig. 105) be a cylinder of equal weight to H, but of double its diameter. The friction will again be to the pressure as B′C′ to A′C′. But A C being common to the two planes, it is shown that the friction of a cylinder of double diameter but equal weight is to that of the first cylinder as B′C′ to B C.

The equation to express the force necessary to overcome this friction when acting at the circumference of the roller, may be represented thus:

Let P=pressure, R=radius, and f=coefficient of friction. Then

$$F=f\frac{P}{R}$$

In a third case, let M (fig. 106) be a cylinder of three inches diameter and six pounds weight; B C represents its friction, and A C its pressure.

Fig. 106.

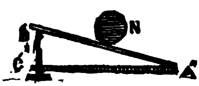

Fig. 107.

Then, let N (fig. 107) be a cylinder of five inches diameter and eight pounds weight, B′ C′ represents its friction and A′ C′ its pressure. By comparing the

lengths of A C with A′ C′, and B C with B′ C′, it will be found that the friction of the three-inch cylinder will be to that of the five inch-cylinder, as $5 \times 6 : 3 \times 8$, or as $5 : 4$.

Axioms.—Wherefore, in experiments with a body rolling over the surface of another, it is found that

1.—*With cylinders of the same substances and equal diameters, the friction is as the pressure.*

2.—*With cylinders of equal pressures and the same substances, but of different diameters, the friction is inversely as the diameters.*

3.—*With cylinders of the same substances but different diameters and pressures, the friction is directly as the pressures and inversely as the diameters.*

Globes.—The angle of repose of a globe is found to be less than that of any other shaped body. Wherefore, the friction of a globe is the minimum of friction with an equal pressure.

The application of unguents to the surfaces of either of the bodies in contact does not diminish this kind of friction in the least degree.

The following table, extracted from Morin's *Notions Fondamentales de Mécanique*, show some of the results of experiments on the friction of various bodies :—

Substances.	Angle of Repose.	Coefficient of Friction.
Oak on Oak, fibres parallel . .	31½°	0·62
,, ,, ,, perpendicular .	28½°	0·54
Oak on Elm, ,, parallel . . .	20½°	0·38
Elm on Oak, ,, ,, . . .	34½°	0·69
Wood on Wood, dry	14° to 26½°	0·25 to 0·5
,, ,, ,, soaped . . .	2° to 11½°	0·2 to 0·04
Metals on Metals, dry	8½° to 11½°	0·15 to 0·2
,, ,, ,, wet and clean .	16½°	0·3
Metals on Oak, dry	26½° to 31°	0·5 to 0·6
,, ,, soapy	11½°	0·2
Leather on Metals, dry . . .	29½°	0·56
,, ,, ,, wet . . .	20°	0·36
,, ,, ,, greasy . .	13°	0·23
,, ,, ,, oily . . .	8½°	0·15
Smoothest and best greased surfaces	1¾° to 2°	0·03 to 0·036

CHAPTER XV.

RIGIDITY OF CORDAGE.

In estimating the mechanical efficiency of systems of pulleys, we have neglected the consideration of the stiffness of the fall, although force is necessary to bend the rope into an arc over the pulley, and again to straighten it after it has passed over.

In determining the amount of resistance due to rigidity,[1] it is necessary to consider the forces applied to act along the centre of the rope, or with a leverage composed of the radius of the pulley and half the diameter of the rope. This is the *effective radius* of the sheave.

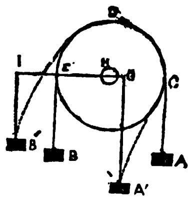

Fig. 108.

In fig. 108, D is a sheave over which the rope C D E passes, to the ends of which equal weights, A and B, are suspended.

The part of the rope C D E is bent into an arc, and has a tendency to retain that form by reason of its stiffness.

If a small additional weight be suspended from A, it will cause the sheave to rotate in the direction E D C. Owing to the rigidity of the rope, the cords will not become straightened immediately upon leaving the sheave, but will take the form shown by the dotted line. From A′ and B′ draw lines, A′ G, B′ I, perpendicular to G I, drawn through the centre of the pin at right angles to

[1] L., *rigidus*, stiff.

the direction of the strain. It is evident that the weight A′ is now acting with the leverage G H, and the weight B′ is acting at the end of the lever H I. Owing to the stiffness of the rope, the leverage of the working part of the rope is diminished, while the loaded part has an increased leverage. If the force applied to A, multiplied by the distance G H, is not greater than the weight B multiplied by the length H I, no motion can ensue; so that unequal weights may be placed at opposite ends of a rope passing over a pulley, and the machine will still continue in equilibrium. It is on this account that while theoretically the pulley is by its action applicable to the purposes of a balance, it is practically of little use in this direction.

The forces opposed to the mechanical efficiency of the pulley, may be classed under two heads—(*a*), those due to the rigidity of the rope; and (*b*), those relating to the weight upon the pulley. As the curvature of the rope over the sheave increases, so the resistance due to rigidity becomes greater; this resistance also increases faster than the radius of the rope.

In winding a rope off a drum upon which it has been coiled, the rigidity is not taken into account.

The table annexed is the result of Morin's calculations from the experiments of Coloumb :—

Radius of Rope.	Circum-ference of Rope.	New Dry Ropes.		Tarred Ropes.	
Inch.	Inch.	A	B	A	B
0·16	1·0	0·32	0·034910	0·41	0·028917
0·24	1·5	1·43	0·078543	1·44	0·065068
0·32	2·0	4·31	0·139640	3·86	0·115668
0·40	2·5	10·31	0·218183	8·64	0·180731
0·48	3·0	21·13	0·314190	17·03	0·260253
0·56	3·5	38·87	0·427643	30·56	0·354233
0·64	4·0	66·00	0·558560	51·05	0·462627
0·72	4·5	105·38	0·706723	80·08	0·585569
0·80	5·0	160·23	0·872750	121·50	0·722925

From these tables it will be seen that the resistance is less for tarred ropes, except those of very small size, than for dry ropes of equal radius.

The following rule for obtaining the resistance due to the rigidity of ropes is in general use: *Multiply B in the preceding table by the weight in pounds, and add the product to A. Divide the sum by the effective radius of the sheave in inches, and the quotient will give the resistance in pounds.*

Thus, if the weight to be raised is 500 pounds, and a 3-inch dry rope is used to lift it, passing round a sheave of five-and-a-half inches radius, the resistance due to rigidity is 29·8 pounds.

The effective radius of the sheave is its real radius plus the radius of the rope given in the table.

The following equation will determine the resistance where $x=$ the weight required to be added to A′ to move the machine. By the principle of the lever we have

$$(x+A')\,r=B'\,(r+b).$$

r expressing the radius of the pulley and $b=EI$. Hence

$$x\,r+A'r=B'r+B'b.$$

Since $A'=B'$; $A'r=B'r$. If these equals be taken from both, we have

$$x\,r=B'b\ \therefore\ x=B'\frac{b}{r}$$

Wherefore, if b is known, the corresponding resistance due to rigidity becomes apparent, since it is only required to consider the leverage of the resistance to be greater than it is by a certain quantity, and this quantity depends upon the curvature of the rope B′ E.

The curvature depends upon four things:—1st, the weight attached, let this $=w$; 2nd, the radius of the rope $=d$; 3rd, the material of the rope $=a$; 4th, the radius of the wheel $=r$.

The determination of the above-mentioned quantity is performed by means of an empirical[1] formula assumed to represent x, viz.—

$$x=\frac{d^{n}}{r}(a+m\,w).$$

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[1] That is one, the truth of which is verified by experiment.

Here the letters m and n represent indeterminate numbers, the value of which, as also that of a, can only be determined by actual experiment. Let four sheaves be taken whose radii are r, r^a, r^b, r^x, and let the rope whose rigidity is to be determined be laid over these in succession, and suspend weights equal to w, w^a, w^b, w^c, and let the weights added to impart motion be x, x^a, x^b, x^c. By substituting these in the above formula, we obtain—

$$x = \frac{d^n}{r}(a+m\,w)$$

$$x_a = \frac{d^n}{r_a}(a+m\,w_a)$$

$$x^b = \frac{d^n}{r_b}(a+m\,w^b)$$

$$x^c = \frac{d^n}{r^c}(a+m\,w^c)$$

From any three of these four equations the value of a, m, and n may be ascertained. We have then—

$$x = w\,\frac{b}{r} \therefore b = \frac{x}{w}\,r;$$

hence—

$$b = \frac{d^n}{w}(a+m\,w).$$

Thus we obtain the increase of leverage equivalent to the resistance due to the rigidity of a rope, its diameter and tension being known.

DIVISION II.

Kinetics.[1]

CHAPTER I.

INTRODUCTORY.

Definition. — Hitherto we have been considering forces in équilibrium, that is to say, that arrangement of forces which tends to keep a body in a state of rest.

It now remains to consider the operation of forces not in equilibrium; or, in other words, those operations which impart motion to the body acted upon.

This portion of our subject is called Kinetics, which may therefore be defined as that branch of Dynamics which treats of the relation between forces and the motions they produce.

Divisions.—This section of Dynamics will be considered under the following heads:—

1. Falling Bodies.
2. The Pendulum.
3. Motion down an Inclined Plane.
4. Projectiles.
5. Motion in a Circle.
6. Central Forces.
7. Impact.
8. Work.

Properties of matter.—Before we are in a position to explain the various forces producing motion, there are certain properties of matter which require to be understood. These are as follows:—

The Quantity of Matter in a body is proportional to its volume and density conjointly. Thus: If A be a body having a volume V and a density D, and B a body hav-

[1] Gr., *kineo*, to move.

ing a volume equal to twice V and a density equal to the half of D, the quantities of matter in A and B are equal. Hence, when the products of the volume and density of bodies are equal, the quantities of matter in the bodies are also equal.

The Quantity of Motion is proportional to the quantity of matter and velocity. Thus: If A be a body having a quantity of matter Q and a velocity V, and B a body having a quantity of matter equal to twice Q and a velocity equal to half V, the quantities of motion in A and B are equal. The quantity of motion is called **Momentum.**[1]

The Change of Momentum is proportional to the mass in motion and the change in the velocity, conjointly.

A ball of lead weighing 10 lbs. and moving with a velocity of 15 feet per second, would strike an obstacle with the same force as a ball weighing 50 lbs. moving at the rate of 3 feet per second. If we have but a small body with which to overcome a great resistance, but can impart to that body great velocity, so as to generate a great momentum, and then suddenly oppose the resistance by that momentum, we produce a force capable of overcoming the resistance. For example, in the pile-driving engine, the *monkey*[2] might lie on the top of the pile for an indefinite time without producing any effect. But by raising it to the top of the *sheers*[3] and letting it fall upon the top of the pile, the momentum acquired by its motion and weight is such that the pile is driven into the earth.

Again, although we could never succeed in pushing a candle through a board, yet, when we impart great velocity to the candle, by firing it from a gun, we can drive it through an inch plank.

The sudden arresting of momentum is called *percussion*;[4] and in our volume on Heat, it will be shown that the motion is changed by percussion into heat.

Kinetic Force.—In kinetics force is measured by a unit which, acting on a unit of mass, produces a unit of acceleration, or generates a unit of velocity in a unit of time.

The measure of kinetic force is, therefore, the quantity of motion produced in a unit of time; and the

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[1] L., *moveo*, to move, moving force. [2] The weight. [3] The scaffolding. [4] L., *percutio*, to strike through.

change in the velocity enables us to compare the magnitudes of different kinetic forces.

The amount of gravitation of various parts of the earth's surface was ascertained by comparing the velocities acquired in a unit of time by the falling of the same body in different localities.

Hence, if different masses, each acted upon by a force, acquire in equal units of time the same velocity, the forces are proportional to the masses, as is the case with the force of gravity on falling bodies.

CHAPTER II.

FALLING BODIES.

Illustrations.—If we throw a stone into the air, the force of gravity acting upon it will eventually cause it to descend to the earth. When bodies of different material fall through the air, they do not usually pass through the same number of units of space in the same time; and bodies of the same material fall through a greater or less number of units of space according to their shape.

Thus, a ball of lead and a piece of paper fall with different velocities. If an ounce of copper be beaten out into a sheet having a surface of one square foot, and another ounce be cast into a bullet, and the two bodies be let fall from the same height at the same instant, the ball will be found to reach the earth before the sheet of metal.

The difference is caused by the resistance opposed to the descent of the body by the upward pressure of the air on the surface opposed to it. This resistance varies with the form and dimensions of the body and with the velocity.

If, however, bodies which fall with different velocities through the air, such as a ball of lead and a feather,

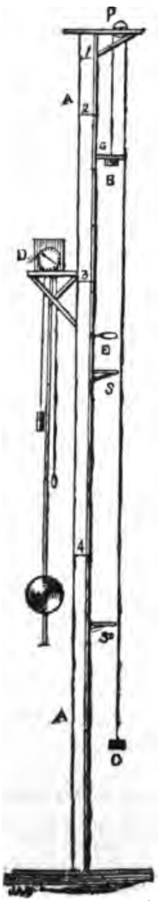

Fig. 109.

numbers, and at the end of that second the body will be $16 \times 9 = 144$ feet from its starting point, 9 being the square of the number of seconds.

The *velocity* at any point is found by multiplying g by the number of seconds from rest. In the above example the velocity at the end of the third second will be 96·6 feet nearly.

The height of a building or depth of a wall may therefore be found by multiplying the square of the number of seconds occupied by a stone in its descent by 16. Thus, if a stone dropped from the battlements of a tower reaches the ground in four seconds $4^2 = 16$ and $16 \times 16 = 256$ ft. = height of tower.

Many watches are constructed to beat quarter seconds, and the intervals between each beat can easily be counted. If the square of the number of quarter seconds be taken, it will give the height. Thus, in the above example, the stone occupied 16 quarter seconds in its descent, and $16^2 = 256$ ft. = the height.

Attwood's Machine.—The law of falling bodies has been verified by means of an ingenious machine invented by Attwood, and called after him *Attwood's Machine.* It consists of an upright frame, A (fig. 109), having a scale attached. On the top of the frame is a pulley P, the pin of which turns on rollers to bring the resistance of friction to a minimum. Over this pulley a thread passes, bearing two equal and similar weights, B and C, which are therefore in equilibrium. G is a small weight capable of being

attached to B at pleasure. If it be placed on B the weight B will descend. The pressure = G, and the weight moved $=B+C=2B$. The acceleration is therefore $\frac{G}{2B+G}$ and this proportion can be made as minute as desired, by making the weight G very small. D is a clock constructed to beat seconds. Let B start from the division 1 of the scale, and let it fall for one second. Let 2 be its position at the end of the first second. If it be again let fall from 1 and stopped at the end of two seconds, it will have arrived at 3. Again let it fall from 1 for three seconds, and it will have reached 4. It will be found that the distance $1...3 = 4\times 1...2$, the distance $1...4 = 9\times 1...2$, and so on, showing the correctness of the rule, that the space is proportional to the square of the time.

Again, E is a ring capable of being moved up and down the bar A. Let it be so placed as to catch G as B passes 2, at the end of one second of time. B will then continue to move with the velocity it had when G was lifted from it. S is a stage also capable of being moved up and down the beam A. Let it be placed so as to catch B one second after it has passed E. E S will then represent the space described by a body moving with uniform velocity for one second. Again, let the stage S be placed at S^2 so as to receive B two seconds after passing E. E S^2 will represent the space moved over by a body in uniform motion for two seconds, and will be found to be double the space E S.

Hence, the velocity acquired is proportional to the time.

Graphic representation of acceleration of gravity.—If we take a length to represent seconds of time, as A B, B C, etc. (fig. 110), and mark off these lengths on a horizontal line, the length of A F will represent the time occupied by the body in its descent. From B, C, D, E, F, draw lines at right angles to A F, representing, according to the scale chosen, the spaces through which a body would fall in each consecutive second.

Thus, B G=16 feet; C H=64 feet; D I=144, and so on. The distance through which the body falls in the

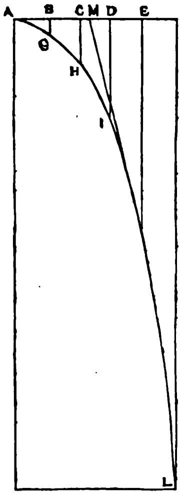

Fig. 110.

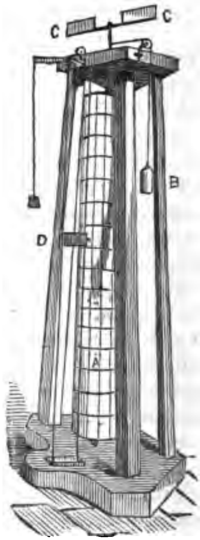

Fig. 111.

first second being multiplied by the numbers 1, 4, 9, 16, gives the length of each consecutive line, B G, C H, etc. The points G, H, I, K, and L will represent the position of the body at the end of each successive second, and the curved line drawn through them will represent the acceleration of the velocity of the falling body.

Morin's Apparatus.—This curve has been marked out by an apparatus invented by General Morin.

It consists of a cylinder, A (fig. 111), caused to revolve about its axis by the descent of the weight B, attached to a cord wound round a horizontal axis, communicating by means of a toothed wheel with an endless screw on the axis of A. The uniformity of the motion is secured

by a fly-wheel and vanes, C C. The cylinder is covered with paper ruled with vertical and horizontal lines. D is a weight sliding between two tightly stretched vertical wires, carrying a pencil, whose point presses lightly against the paper on the cylinder A. The weight is supported by a cord, and can be detached at pleasure. When the cylinder is stationary, the line marked by the descent of D will be a vertical line; while, if the weight is stationary and the cylinder revolves, the line will be horizontal. If the force of gravity were a uniform force, when the weight descended during the revolution of the cylinder, the line marked on the paper by the descent of the pencil would be the diagonal of the squares marked on the paper cylinder.

As the force of gravity is a uniformly accelerating force, the line described by the descent of the weight is a curved line, and is found to be a parabola. The joint effect of the fall of the weight and the revolution of the cylinder produces a resultant path coincident with that described by a body projected with a uniform velocity and allowed to fall by the action of gravity.

The horizontal velocity of the cylinder for each unit of time is determined beforehand; and experiments prove that the falling weight at the end of a certain time is at a point on the vertical line drawn from the point at which it would have arrived, if it had moved horizontally only. The distances of these points from the horizontal line drawn round the cylinder at the height from which the weight fell, increase as the squares of the times, *i.e.* as the numbers 1, 4, 9, 16, &c. This was the result arrived at by the curve of acceleration (fig. 110); and the coincidence of these two curves confirms the theory of the second law of motion.[1] The ratio of the perpendicular velocity of the falling body at any point to the horizontal velocity of the cylinder, is determined by drawing a tangent to the curve at that point, and producing it to

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[1] Page 41.

meet the line representing the horizontal velocity, and dividing the perpendicular distance of the point from the horizontal line by the length of the line intersected between the tangent and the perpendicular. Thus, in fig. 109, the ratio between the perpendicular velocity of the particle at the point M is found by drawing the line K M tangentially to the curve A K L at the point K, and dividing the perpendicular K E by the distance A M.

When a body is projected vertically upwards with a certain velocity, it rises with a negative acceleration for a number of seconds, found by dividing the velocity by 32, or the velocity of a body falling for one second. The height to which it will rise is found by dividing the square of the velocity by 64 or twice the acceleration of a falling body in one second of time.

Thus, if a ball be thrown vertically upwards with a velocity of 72 feet per second, it will rise $72 \div 32$ seconds, or $2\frac{1}{4}$ seconds, and will ascend to $72^2 \div 64$ feet, or 81 feet, and will descend in a time equal to that occupied by its ascent and reach the ground with a velocity equal to its starting velocity.

CHAPTER III.

THE PENDULUM.

Definition.—The pendulum[1] is an instrument used to measure time. Any body capable of moving freely about a fixed axis is a pendulum, and will assume a position having its centre of gravity directly below the point of suspension.

Varieties of Pendulums.—Pendulums are of two kinds:—

1st. Simple pendulums, consisting of a single heavy particle suspended from an axis; and—

2nd. Compound pendulums, consisting of a number of particles suspended from a single axis.

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[1] L., *pendeo*, to hang; *pendulum*, a small suspended body.

Simple pendulums have really no existence, for the weight is always more than a single particle, and the connection between it and the axis possesses weight likewise. It is convenient, however, to suppose them to exist, in order to simplify the proofs of the laws governing pendulums.

I. The Simple Pendulum.—The simplest form of pendulum is a heavy weight, W (fig. 112), suspended from a fixed point, P, by an inextensible rod or thread. By the force of gravity, the weight, which is called the *bob*, will assume a position in the vertical line from the fixed point. If, however, it be displaced from the vertical

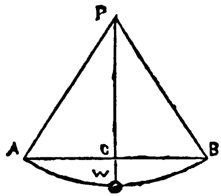

Fig. 112.

into the position P A, and then be released, it will swing to a position, P B, on the opposite side of the vertical. Arrived at B, it will again fall back to W, and move on to A. If there were no resisting forces, such as friction and the resistance of the air, the distance C B would be equal to C A, and the motion would continue for ever. In consequence, however, of the presence of these passive forces, each successive swing extends over a smaller arc than the previous one, until the bob comes to a state of rest.

The swinging is called *oscillation*[1] or *oscillatory motion*. The angle of the sector[2] A P B, through the arc of

[1] L., *oscillare*, to swing. [2] L., *sectus*, cut. A sector is the space cut off by two radii and an arc of a circle.

which the bob moves, is called the *amplitude*[1] *of oscillation.*

The velocity with which the bob will descend the arc A W, is proportional to the height of C above W. The length of the time of one oscillation is therefore double the time required to descend C W; and the time of descent from A to W equals the time of ascent from W to B; and the successive oscillations take place in equal times. The amplitude of oscillation is generally small, and the diminution of the arc has but little effect upon the time of each oscillation.

Isochronism.—This property of the pendulum of swinging to and fro in equal times, is called *isochronism,*[2] and may be thus illustrated: If we roll a bullet down the inside of a hemispherical bowl, it will ascend the opposite side, and oscillate in arcs of a vertical circle. When these arcs are small, the times of each oscillation will be equal; but when the arcs are large, the times are unequal. A circle is therefore only isochronic through a small arc for a body acted on by gravity. If the bowl be cycloidal[3] the times of oscillation will be equal, whatever the amplitude of oscillation may be. The cycloid is therefore called the *isochronic curve.*

The time of oscillation of a simple pendulum is equal to the time of falling through half the length of the pendulum, multiplied by 3·1416. For, $s = \frac{1}{2} g\, t^2 \therefore t = \sqrt{\frac{2s}{g}}$. Then, $2s = l \therefore t = \sqrt{\frac{l}{g}}$ or the time of falling through half the length. The time of a single vibration is, therefore, given by the formula $T = \pi \sqrt{\frac{l}{g}}$ . . . ($\pi = 3{\cdot}1416$ $\therefore$ $g = 32{\cdot}2$). The time of vibration is, therefore, proportional to the square root of the length; for the time of vibration being doubled by quadrupling the length of the pendulum, the square of the number of vibrations is inversely proportional to the length, or—

$$T : t :: \sqrt{L} : \sqrt{l}; \text{ or, } T^2 : t^2 :: L : l.$$
$$N^2 : n^2 :: l : L; \text{ or, } N : n :: \sqrt{l} : \sqrt{L}.$$

[1] L., *amplus*, large. [2] Gr., *isos*, equal; *chronos*, time. [3] A cycloid, is the curve traced by a point on the circumference of a circle rolling on a straight line.

Variation of the force of gravity.—The force with which the bob will descend from C to W depends upon the force of gravity. Since the force of gravity varies with the latitude of the place, the velocity acquired in descending C W also varies with the latitude. A pendulum vibrating in a certain number of units of time in London, would therefore vibrate in a less number of units of time at the equator, because the force of gravity there is less.

The isochronism of the pendulum is utilized in measuring time. The force of gravity of the place is ascertained, and from this the length of pendulum required to beat seconds is calculated. The length of the seconds pendulum in London is 39·139 inches, at Spitzbergen 39·2144 inches, and at the Island of Rawak, near the equator, only 39·0144 inches.

By means of pendulums, the fact of the centre of gravity of the earth not coinciding with its geometric centre was made known; for it was found that pendulums vibrating in equal times on equal parallels of latitude were not always of equal lengths in different meridians.

II. **Compound Pendulums.**—We have before shown that simple pendulums are practically unknown; but we have treated of the properties of pendulums under the head of simple pendulums for convenience. All pendulums are compound, for the bob is of appreciable size and the thread possesses weight. All the particles of a pendulum, if it were suddenly to break up into its component particles, would vibrate in times proportional to their distances from the centre of oscillation. Those nearer the point of suspension than the C. G. of the bob would vibrate quicker, and those further from it would vibrate slower than before. Only that particle whose position is in the C. G. of the bob would vibrate in the same time as before the separation. If every particle could be squeezed into this one particle, a pendulum called the *equivalent simple pendulum* would be formed. The point vibrating

at the same rate as the compound pendulum, is called the *centre of oscillation.*

The length, therefore, of a compound pendulum is the distance between its axis and centre of oscillation. The rule for determining this length was discovered by Huyghens.

Fig. 113.

A B (fig. 113) is a pendulum vibrating about its axis, C. If it be next suspended from its centre of oscillation, D, it will vibrate in the same time as before. To find the length of a pendulum, as A B, let it be first suspended from C and its vibrations noted. Then suspend it from a point, D, so that its times of vibrations are the same as when suspended from C. The distance C D is the length of a simple equivalent pendulum whose vibrations are equivalent to those of the compound pendulum A B.

Isochronism of the Balance-wheel. — The balance-wheel of a watch is rendered isochronous in its motion by fixing to its axis one end of a fine spiral steel spring, and the other end to some fixed point in the frame which supports the wheel. When the balance-wheel arrives at the end of its swing to the right, the recoil of the spring imparts to it a momentum sufficient to carry it back the same distance to the left of its original position. Thus its motion becomes isochronous for reasons similar to those of the isochronism of the pendulum.

Laws of the Simple Pendulum.—The following properties possessed by all simple pendulums have been called, the *Laws of Simple Pendulums.*

1. *The motions of pendulums are isochronous.*
2. *Their times of oscillation are independent of the weight of the bob.*
3. *Their times of oscillation will vary as the square roots of their lengths.*
4. *Their times of oscillation will vary inversely as the square root of the force of gravity at the place.*

A pendulum required to beat half seconds, therefore, will be one-fourth the length of a seconds pendulum, and to beat thirds of seconds one-ninth of that length.

The length of the seconds pendulum has furnished a standard of length which is invariable and capable of being recovered at any time. By Act 5, Geo. IV., the yard is defined to consist of 36 parts, of which there are 39·1393 in the length of a pendulum vibrating mean seconds at London, in vacuo at a temperature of 62°F.; 39·1393 inches being the length of a seconds pendulum in London.

CHAPTER IV.

MOTION DOWN AN INCLINED PLANE.

Explanation.—Motion down an inclined plane may be made to approximate more or less that of a falling body, by reducing or increasing the proportion between the base and height of the plane. This is evident, since the distance between the two points A and B at the ends of the base, may be taken indefinitely near to each other. When the base is equal to the height, the body will move over a space equal to the diagonal of the square having the height of the inclined plane as one of its sides.

We have shown, in speaking of the inclined plane, that if a force acts in a direction parallel to the plane, equilibrium is produced when the power is to the weight as the height of the plane is to its base. In other words, if the power equals the weight multiplied by the height, and divided by the length of the plane, the body will be at rest on the plane. The weight, multiplied by the height and divided by the length of the plane, is the force tending to produce motion down the plane; and this force being uniform, the motion will be uniformly accelerated. This acceleration being due to gravity,

we obtain the two following theorems of motion on an inclined plane.

1 *The velocity acquired by a body in running down an inclined plane, is equal to the velocity acquired in falling down the height of the plane; and,*

2. *The time of running down any chord of a vertical circle, drawn from the highest point, is constant; since all chords are the lengths of inclined planes, having the height of the circle for height, and the perpendiculars from the vertical diameter to the extremity of the chords for bases.*

CHAPTER V.

PROJECTILES.[1]

Definition.—A projectile is a body thrown, and the path it describes is called the *trajectory.*[2]

Direction.—When a body is thrown vertically upwards, it will ascend in a straight line, and descend to the place from which it started. When the direction is inclined to the horizon, the trajectory is a curve.

It has already been demonstrated in speaking of Morin's apparatus, that the path of a body projected horizontally and acted on by the force of gravity, is a parabola.

When the direction is oblique, the nature of the path is also a parabola, as may be seen by reference to fig. 114.

Let A B be a horizontal and A C a vertical line, drawn from the point of projection A; and let A D be the

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[1] L., *projicio*, to throw forward.
[1] L., *trans*, over; and *jaceo*, to throw.

direction of the projection. If there were no force of gravity, A D would be the trajectory; but since the force of gravity constantly draws the projectile towards

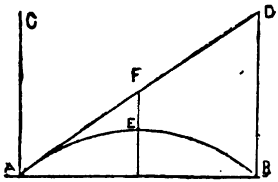

Fig. 114.

the earth, its path will be between A D and the horizontal A B.

Now, as the force of gravity acts the same whether the body be in motion or at rest, a projectile will fall below the line of projection the same distance in the same time as another body of equal weight would fall if started from a state of rest at the same time as the projectile. If therefore we set off from the line of projection vertical lines proportioned to the distance through which the body would fall at the end of successive times, we obtain the position of the body at the end of those times.

But this is the construction required to produce the curve called a parabola. The path of a projectile is therefore a *parabola*.

For example, if the body be projected with a velocity sufficient to carry it in a given time to F (were there no gravity), it will be found at the end of the time, owing to the action of gravity, to have arrived at E in the vertical from F; and at the end of the time in which it would have arrived at D, were there no force of gravity, it will have been brought to the earth at B; and the curve A E B has been shown to be a parabola.

Range.—To determine the greatest height to which a projectile will rise, the velocity at starting is resolved into two components, the vertical velocity, and the horizontal velocity; and the greatest height is found by dividing the square of the vertical velocity by twice the acceleration due to gravity.

If a shot be fired with a vertical velocity of 500 feet per second, $500^2 = 250,000$, which, divided by 64, $= 3906$, the height to which the projectile will rise.

The range on a horizontal plane is found by dividing twice the product of the vertical and horizontal velocities by the acceleration of the velocity of a falling body.

Thus, if a bullet fired obliquely upwards have a vertical velocity of 300 feet per second and a horizontal velocity of 500 feet per second, $300 \times 500 \times 2 = 300,000$, which, divided by 32, $= 9375$, the range in feet.

The range will therefore be greatest when the angle of projection is 45 degrees; for, by the last example, the total velocities are equal to 800 feet per second; and if the angle had been 45°, or half a right angle, the horizontal and vertical velocities would have been equal to each other, or 400 feet per second, and $400 \times 400 \times 2 \div 32 = 10,000$.

In the theory of projectiles no notice is taken of the resistance of the air, which is so considerable as to render the application of the theory to gunnery useless; since the trajectory approaches the earth much sooner than would be the case if it were actually a parabola. The resistance of the air is generally a large multiple of the weight of the projectile.

CHAPTER VI.

MOTION IN A CIRCLE.

Explanation.—A particle in motion under the influence of a single force moves along a straight path. If two or more forces act upon it, their joint effect is to alter either the direction or velocity of the motion, or both. If the forces act along the same path as the particle, their joint effect is to accelerate or retard its motion, according as to whether their direction is the same as or opposite to that of the particle. If the forces act at an angle to the path of the particle, their joint effect is to change the original direction of the particle into another direction, which may be either a straight line or a curve. Under certain conditions the curve is a circle.

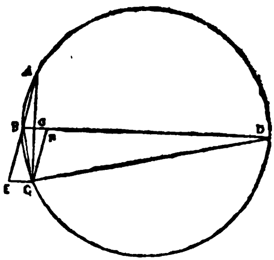

Fig. 115.

Uniform Motion in a Circle.—Let A, B, C, D (fig. 115) be points in the circular path described by the particle. Within the circle describe a polygon, whose sides, A B, B C, nearly approximate to the circle. Let A B be the path described by the particle in the first unit of time. A B will therefore represent the velocity during that time. Produce A B to E, and make B E = A B. If no other force acted upon the particle describing the path A B in the first unit of time, its path in the second unit of time would be B E. At B let another

force act upon the particle in the direction B D, at right angles to A B and equal to B F. This force will not alter the velocity, but only the direction of the motion of the particle, which at the end of the second second will arrive at C, instead of at E, and will describe the path B C. Complete the parallelogram B E C F, and join C D. B F will then represent the deflecting force, and B E the original force. The force B F is called the *centripetal force*,[1] and is equal to the force C E, which tends to keep the particle away from the centre, and is therefore called the *centrifugal*[2] force.

To determine the amount of the deflecting force, join A C. The two triangles D B C and B G C are similar, therefore D B : B C :: B C : B G $\therefore$ $BC^2 = DB \times BG$. Let r = the radius, and f the centripetal force. $DB = 2r$, and $BG = \frac{1}{2} BF = \frac{1}{2} f$, and $BC = v$. By substituting these values we have

$$v^2 = 2r + \frac{f}{2} = fr \therefore f = \frac{v^2}{r}.$$

The centrifugal force is, therefore, equal and opposite to the deflecting or centripetal force, and consequently directly proportional to the square of the velocity, and inversely proportional to the radius of the circle described. In other words, the centrifugal force may be found by multiplying the weight of the body by the square of its velocity, and dividing by the acceleration of gravity and the radius of the circle.

An illustration of centrifugal force is shown in a stone whirled round by a string. If the stone be 1 pound weight, the string three feet long, and its velocity 24 feet per second, by the above formula, 1 lb. $\times 24 \times 24 \div 32 \times 3 = 6$ lbs., the centrifugal force acting on the stone, which is counteracted by the string, representing the centripetal force, which is also equal to 6 pounds.

The centrifugal forces of two unequal bodies moving with equal velocities at different distances from the centre, are to one another as their quantities of matter multiplied by their distance from the centre.

Effects of Centrifugal Force.—If a globe be turned about a fixed axis, each point in it describes the circumference of a circle, whose plane is perpendicular to the

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[1] L., *centrum*, the centre; *peto*, to seek. [2] L., *centrum*; and *fugo*, to fly.

axis, and whose radius is equal to the distance of the point from the axis.

All points on the globe which describe unequal circles in equal times have velocities which are *inversely* proportional to the radii of the circles they describe.

The velocity of the particles composing the earth is greatest at the equator, and diminishes as we approach the poles, where the motion is nothing. The tendency of the particles to fly from the centre is, therefore, greatest at the equator; and if these particles were free to move, they would fly off. We have every reason to believe the earth was once in a fluid state,[1] when the particles would possess a much smaller amount of cohesion than at present. The motion of the earth would cause the particles at the equator to move from the centre, while the particles at the poles would approach the centre.

This may be approximately illustrated by the following apparatus (fig. 116).

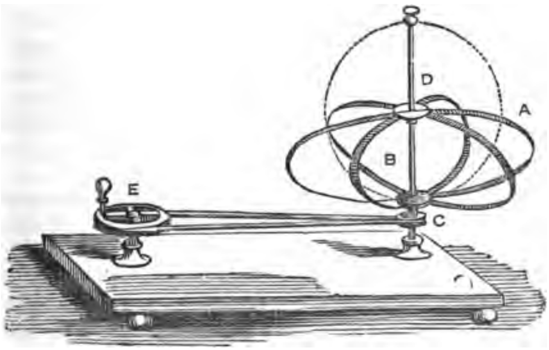

Fig. 116.

A, B, are two circular hoops of thin steel, fixed to the

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[1] See Geology, p. 9.

centre at C, but capable of moving up and down the axis at D. E is a multiplying wheel, by which rapid motion may be communicated to the hoops.

As the velocity increases, the particles composing the hoops, being acted upon by centrifugal force, seek to fly off from the axis, but are prevented from so doing by the force of cohesion existing between them. The resultant motion of all the particles will cause the diameter of the hoops to extend in a direction perpendicular to the axis of rotation. The circumference of the hoops remaining unaltered in length, the diameter along the axis will decrease to allow for the increase of diameter occasioned by the centrifugal force.

The shape of the hoops when in motion is called an ellipse; and the figure generated by an *ellipse* moving about its shorter axis is called an *oblate spheroid*, which is the shape assumed by the earth.

The variation in the shape of the earth from a perfect sphere is one of the causes of variation in the force of gravity; and as the centrifugal force is greatest at the equator, and diminishes towards the poles, the tendency of a body to fly off is greatest at the equator; and as this tendency is opposed to the attraction of gravitation, its effect must, of course, be deducted from it. The force of gravity is, therefore, least at the equator, because it acts upon bodies which are farther away from the centre, and have greater centrifugal force.

It is easy to conceive that if the velocity were great enough, a revolving globe would assume a disc-like shape by the polar diameter becoming infinitely short. This effect is utilized in glass-blowing, where the blower takes up on the end of his blow-tube a ball of molten glass, and by whirling it round causes the particles to fly from the centre and thus form a circular plate of glass.

Centrifugal force is also illustrated by the trundling of a mop. The woollen fibres of the mop are fixed to the handle; but the water, being free to move, flies off.

Carriages on Curves.—The neglect of the effects of centrifugal force often causes carriages to overturn when rapidly driven round a curve. The weight of the carriage acts in the direction of the vertical, whether the carriage is in motion or at rest. When a motion round a centre is given to the carriage, a centrifugal force is generated, acting from the centre of gravity of the carriage at right angles to the axis of the curve.

Thus: If A (fig. 117) be the centre of gravity of the

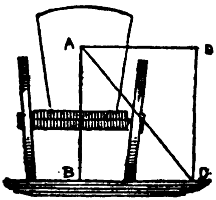

Fig. 117.

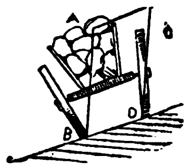

Fig. 118.

carriage, its weight will act in the direction of the line A B. If the carriage be driven sharply round a curve, the centrifugal force A C will be generated, and the resultant of these two forces will be A D, falling outside the wheels, and the carriage will consequently be upset.

If the plane on which the body moves be inclined downwards towards the centre of the curve (fig. 118), the resultant will fall within the base, and the equilibrium will be stable. For this reason, in sharp railway curves, the outer rail is made higher than the inner. For the same reason a skater, when turning round, leans toward the centre of the curve he is describing.

The Governor.—One of the principal uses to which centrifugal force is applied, is to regulate the speed of machines, especially steam engines, where the resistance is variable. The mechanism exemplifying this, called a *governor*, is shown in fig. 119.

A B, A C, are two arms, capable of moving on axes at

A, and having heavy balls, B C, attached. The axes of the arms are fixed at A to the shaft A S, which revolves with the engine. D F, E F are two arms hinged

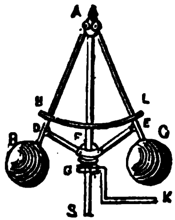

Fig. 119.

at D and E to the arms A B and A C. Their lower ends are hinged at F to a collar, G, capable of moving up and down the shaft A S. To G a rod, K, is attached, communicating with the valve which admits steam to the cylinder. When A S revolves, the arms B C fly outwards and draw up the collar G. The valve at the steam pipe, called a *throttle valve*,[1] is so arranged as to admit sufficient steam to keep the engine at a certain speed when the collar is a little raised. When additional resistance is placed on the engine, the speed diminishes, and the balls fall, the collar being lowered, and thereby admitting more steam into the cylinder. If part of the resistance be taken off, the speed increases, and the balls, flying from the centre, raise the collar G, which cuts off part of the steam. The effect is, therefore, to so regulate the proportion of steam to the resistance, as to keep the engine working at a nearly uniform speed.

[1] That is, a valve to throttle or choke the steam.

CHAPTER VII.

CENTRAL FORCES.

Explanation.—When a number of forces acting on a body tend to move it towards or away from a fixed point, they are called *central forces*, the point being called the *centre of force*. If forces tending towards a fixed point continually act upon a body, their effect is to move it with an accelerating velocity up to the fixed point; but if the forces acting on the body are not in the direction of the fixed point, and are not continuous in their action, the body will not move towards the centre of force, nor will the velocity be accelerated.

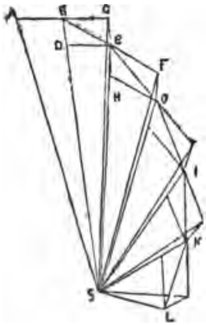

Fig. 120.

Suppose a body acted upon by a force moves with a uniform velocity sufficient to carry it from A to B (fig. 120) in one second of time. If no other force acts upon the body, it will arrive at C at the end of the second second. B C being equal to A B. But suppose a force which would carry the body from B to D towards the centre S begins to act upon it at B. We then have two forces, B C and B D, acting on the point B; and their

resultant will be changed from B C to B E, and the velocity of the body will be the diagonal of the parallelogram B E, drawn with those two forces as adjacent sides. The body, if free to move, subject only to the original force, will arrive at F at the end of the third second, E F being equal to B E. But the central force B D still acting upon it, draws it from E to G. In like manner the position of the body at the end of successive units of time will be found to be in the points H, I, K, L.

Because C E is parallel to C B, the triangle S E B is equal to the triangle S C B, which is equal to the triangle S B A.

Again, because F G is parallel to E H, the triangle S G E is equal to the triangle S E F, which is equal to the triangle S E B; and, therefore, also to the triangle S B A, and the lines S A, S D, are radii vectores.[1] Hence, *the triangles formed by the radii vectores and the path of a body acted on by a central force, are all equal to each other; and if the units of time are taken indefinitely small, the path will be a curve.*

This equality of triangles may therefore be expressed by saying that the *radii vectores of the curve mark out equal areas in equal times;* and that *in different times, the areas marked out by the radii vectores are proportional to the times.*

Hence, if a body move in a curved path, so that the radius vector, drawn to a fixed point, marks out areas proportional to the times, it is acted on by a central force whose direction is towards that fixed point.

Laws.—From these theorems, the following results are mathematically demonstrable:—

1. If a body be acted on by a central force at a fixed point, its velocity at any point in its path, is inversely proportional to the perpendicular drawn from the fixed point to the tangent to the curve at the point under consideration.

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[1] L., *radius*; and *vector*, a carrier.

If the velocity be uniform, all these perpendiculars are equal, being radii of a circle. Hence, the path of a body moving with uniform velocity, and, acted on by a central force, must be a circle, the fixed point being its centre.

2. If a body describe an ellipse, when acted on by a force tending to the focus of an ellipse, the velocity is *inversely proportional to the square of the distance.*

3. If the path of the body be an ellipse, and the force acts towards the centre of the ellipse, the velocity is *proportional to the distance.*

These laws were first proved by Newton and Kepler. Kepler's Laws define the motions of the planets to be in ellipses,[1] the sun occupying one of the foci, and that the radius vector of each planet marks out equal areas in equal times. We may, therefore, conclude that the motions of the planets are caused by central forces.

CHAPTER VIII.

IMPACT.

Laws, etc.—One of the fundamental laws of matter is, that "*Two bodies cannot occupy the same space in the same time.* When two bodies in motion come into contact, reaction commences, to prevent any particles of the one from occupying the same space as any particles of the other.

The centre of impact[2] is the point at which the bodies touched at first; and the force radiates from this centre on a gradually-increasing surface of contact. If the bodies are elastic,[3] this area will attain a maximum, and then gradually diminish until it comes to nothing. This process of expansion and contraction of area is per-

[1] Phys. Geog., p. 10. [2] L., *in*, upon; and *pango*, to strike.
[3] Gr., *elaunō*, to drive.

formed in an inappreciable unit of time. The principal laws of impact are the following:—

1. *The common velocity of two non-elastic bodies after impact, when they both move the same way, is found by dividing the sum of the products of each body into its respective velocity, by the sum of the bodies.*

2. *When the bodies meet each other, divide the difference of the products of each mass into its velocity, by the sum of the bodies, for the common velocity after impact.*—TOMLINSON.

Kinds.—Impact is of two kinds: *direct*, when the bodies move in the same line before and after impact; and *oblique*, when the bodies, after impinging, move in different lines.

The true action which takes place during direct impact may be thus explained. When two bodies, A and B, are in motion, and A overtakes B, both will be compressed so long as A moves faster than B; and when the velocities of each become equal, the compression will cease.

The time during which the velocities are unequal, is called the *period of compression;* and in this period the motions are changed until all relative velocity is destroyed. If the bodies are inelastic, they will afterwards move together as one body.

If the bodies are elastic, the period of compression is followed by a *period of restitution*, when a force called *elasticity* is brought into play, by which the original forms are restored to the bodies, and a mutual pressure is exerted by each body until impact ceases, and the bodies are separated with a relative velocity. In this case A loses momentum during compression, and during expansion also. The period of compression corresponds with the period of expansion of area, and the period of restitution is equal to the period of contraction of area of impact.

Moduli of Elasticity.—By experiment, it is found that the momentum lost by A and gained by B during

the period of compression bears a constant ratio to the momentum lost by A and gained by B during the period of restitution, for the same materials; and this ratio is called the *modulus*[1] *of elasticity*.

A body is inelastic when its modulus is zero; is imperfectly elastic when the modulus is between zero and 1; and is perfectly elastic when the modulus is 1.

The elongation which a force can produce in a body depends upon the nature of the body, its section, and the force itself. The difference of length is found to be directly proportional to the force, and inversely to the section.

If a bar be F feet long, and its sectional area S; and if by a strain of P lbs. its length is increased L inches, it is found that $L : F :: \frac{P}{S} : E$, E being the modulus of elasticity for the material. Hence, if $\frac{P}{S} = E$, L would equal F; or, in other words, if the modulus of elasticity were applied to each square inch of section, the length of the bar would be doubled.

"*The modulus of elasticity is, therefore, that strain per square inch of section of a body which would double its length if its elasticity remained perfect.*"—TWISDEN.

TABLE OF MODULI OF ELASTICITY.

Material.	*Modulus in lbs.*	*Material.*	*Modulus in lbs.*
Cast iron bars	17,000,000	Copper Wire	17,000,000
Wrought "	29,000,000	Oak	1,450,000
Cast Brass	8,930,000	Fir	1,330,000
Hard Steel	29,000,000	Elm	700,000

[1] L., *modulus*, a little measure.

CHAPTER IX.

WORK.

Definition.—The efficacy of a force to overcome resistance is called *work*. The work done by a force depends upon the weight lifted and the space through which it is lifted, conjointly. It is therefore equal to the product of the weight raised, by the height through which it is raised.

Unit of Work.—To estimate the value of work, some unit of measurement must be adopted. The usual unit of work is the *foot-pound;* that is to say, a weight of one pound lifted one foot high. Consequently, if 200 lbs. be raised 12 ft. high, the work done is 2400 foot-pounds. When force is applied to any machine, the work done at the point of application of the force is exactly equal to the work done at the opposite extremity of the machine, friction and other passive forces being neglected.

Horse Power.—In estimating the work done by steam engines, the unit of work is compared to the work done by a horse working eight hours a day. It is proved that a horse could raise 33,000 lbs. one foot high per minute; 33,000 foot-pounds is therefore called a horse power, and an engine of 12 horse power is one capable of exerting a force of 396,000 foot-pounds per minute.

If W be the work done, P the weight raised in lbs., h the height in feet, N = number of workers, w = work done by each in one unit of time, and T = the number of units of time; then,

$$W = wNT \therefore wNT = Ph.$$

Euler's Law.—Mechanical work is generated by the union of a continual pressure with a continual motion. Euler's law, the result of numerous experiments, gives the relation between the velocity and the resistance, in order that a maximum of work may be produced by a minimum of force. This law states that:—*The resistance should be about four-ninths of that which would exactly counteract the power; and the velocity of the point of*

application of the power should be one-third of its greatest velocity.

A machine will therefore be most economically worked when it moves at one-third of its greatest speed, and lifts four-ninths of its load weight.

Energy.[1]—By energy is meant the inherent power or a body or machine to perform work. Thus, a body in motion is capable of doing work by reason of its motion. Again, a watch-spring, when wound up, is capable of performing work by reason of its condition.

The former of these kinds of energy is called *Kinetic*[2] energy, or *Vis Viva;*[3] and the latter is called *Potential*[4] energy.

Kinetic, or Actual, Energy is the amount of work a moving body is capable of doing at any instant. This amount is constant, whatever be the direction of the motion.

If a stone be thrown vertically upwards, the force exerted in throwing it is opposed to the force of gravity, and the stone will ascend with decreasing velocity until its motion upwards ceases. It is evident that its kinetic energy is greatest at starting. Suppose it starts with a velocity which would carry it eighty feet high. At starting, there will be eighty units of kinetic energy in it. When it is sixty feet high, there will be only twenty units of kinetic energy in it; and when it is eighty feet high, its kinetic energy will be exhausted.

The height to which a body, started with a given velocity, will rise, is found by dividing the square of the velocity by twice the acceleration due to gravity; and the height in feet multiplied by the weight of the body will give the number of units of work in the body at starting.

The kinetic energy of a body is, therefore, measured by the product of the weight by the square of the velocity divided by twice the acceleration due to gravity.

Potential Energy.—When the stone has arrived at its

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[1] Gr., *en*, within; and *ergon*, work. [2] Gr., *kinetos*, moving.
[3] L., *vis*, force; and *viva*, living, as opposed to still force.
[4] L., *potens*, able.

greatest height, its *vis viva* is exhausted; but it possesses another kind of energy due to its changed position in relation to the earth. Its kinetic energy has been converted into an advantage of position called *Potential* energy. The sum of these two energies in a body is, therefore, constant, the potential energy increasing as the kinetic energy diminishes.

If 100 units of kinetic energy are required to raise a body to a certain height, as a stone to the top of a wall, 100 units of potential energy will take the place of the units of kinetic energy when the stone is at the top of the wall, and is consequently in a condition to develop 100 units of kinetic energy. The stone continues in this condition as long as it remains in the same position, but no sooner does it leave the top of the wall to fall to the ground, than kinetic energy is again developed at the expense of the potential energy.

In winding up a watch, we use kinetic energy in coiling the spring, which, when, coiled possesses a potential energy to uncoil itself exactly equal to the kinetic energy employed in winding it up.

When a body falls, the work accumulated is the amount of work which would raise it to the height from which it has fallen; or, in other words, the mass multiplied by the height of the fall.

Thus, if a body of 20 lbs. fall from a height of 60 feet, the work accumulated is 1,200 foot-pounds.

If the velocity of a body is known, we can calculate the height from which it should fall to obtain that velocity, since $v^2 = 2gs$. s therefore $= \frac{v^2}{2g}$. The work accumulated in one unit of mass

$$= 1 \times \frac{v2}{2g} = \frac{v2}{2g} \text{ foot-pounds.}$$

If m be the mass and v the velocity of a body, its kinetic energy

$$= \frac{mv^2}{2g}.$$

As the pound is generally considered the unit of mass, the second the unit of time, and the foot the unit of space, the absolute unit $= \frac{1}{g}$ foot-pounds. The mass $= m = \frac{m}{g}$ foot-pounds, and the work accumulated $= \frac{1}{2}mv^2$.

QUESTIONS.

KINEMATICS.

1. A line half an inch long represents a velocity of 3 feet per second. What velocities are represented by lines whose lengths are 3 in., $2\frac{1}{4}$ in., and $4\frac{3}{8}$ in., respectively?

Ans. 18 ft., 13 ft. 6 in., and 26 ft. 3 in., per second.

2. If a body moves at the rate of 7 feet per second, what distance will it travel in 16′ 35″? *Ans.* 2391 yards, 2 ft.

3. If the above velocity is represented by a line $\frac{1}{8}$ in. long, what length of line will show the position of the body at the end of the 11th minute? *Ans.* 6 ft. $10\frac{1}{2}$ in.

4. A body starting from A, has at B, which is 2 in. from A, a velocity of 20 ft. per second. Where must a point C be placed to indicate that the same velocity has been acquired in half the time?

Ans. 1 in. from A.

5. A body has a velocity of 600 miles an hour: Express the same velocity in feet per sec. *Ans.* 880 ft. per sec.

6. In 20 minutes a body has moved over a distance of 586 yds. 2 ft. What distance will it travel in 3 hours 6 min. 40 sec.?

Ans. 3 miles 195 yds. 1 ft. 8 in

7. If the above velocity is represented by a line 4 in. long, what. length will a line 27 ft. 2 in. represent?

Ans. 81 miles 880 yds.

8. A body moves as follows: 25 min. at 30 miles per hr.; 30 min. at 40 miles per hr.; 75 min. at 15 miles per hr.; 7 min. 30 sec. at 24 miles per hr.; 52 min. 30 sec. at 56 miles per hr.; 10 min. at 3 miles per hr.; and 20 min. at $6\frac{3}{4}$ miles per hr. What distance does it travel in the whole time, and what is its mean velocity?

Ans. 106 miles, at $42\frac{8}{10}$ per hour.

9. A body has a mean velocity of 40 miles per hr., and is in motion for 30 min., but for the first 10 min. it moves at the rate of 36 miles per hr. At what rate must it afterwards move to arrive at its destination at the end of the time? *Ans.* 42 miles per hr.

10. A body moves 100 yds. in 5 sec. What is its velocity per hr.? *Ans.* 40 miles 1600 yds.

11. A steamer moves 20 miles per hr. through the water. The tide against her is 5 miles per hour, and a passenger is walking towards the stern at the rate of 4 miles per hr. What is his resultant motion?

Ans. 11 miles per hr. in the direction of the vessel's head.

12. A man is rolling a cask up hill at the rate of 6 miles per hr. He rolls it up for 10 min. and then rests 15 min., during which time the cask rolls back at the rate of 4 miles per hr. He then rolls the cask 30 min. and rests for 10, during which time the cask rolls back at the rate of 12 miles per hr. What is his mean rate of progress up the hill? *Ans.* 1½ miles per hr.

13. A train travels 100 miles in 2 hrs. What velocity is that in ft. per sec.? *Ans.* 73 ft. 4 in. per sec.

14. A and B start from two points, C and D. A reaches E, midway between C and D, in 2 hrs. and meets B at F an hr. after, and arrives at D in 4 hrs. If A walks 6 miles an hr., (*a*) what distance is D from C? (*b*) how far is F from D? and (*c*) when will B reach C? *Ans.* (*a*) 24 miles, (*b*) 6 miles, (*c*) 12 hrs.

15. Give a graphic representation of the motion of the body in question 8.

16. Draw the line representing the following motions, on a scale of ¼ in. = 1 ft., and ⅛ in. = 1 sec. 4 ft. in 2″, 3 ft. in 1″, 7 ft. in 3½″, 5 ft. in 2″, 8 ft. in 1½″.

17. Compare the velocities of two bodies, one of which moves over 1 ft. 5 in. in 17″ and the other over 262 ft. 6 in. in 21″. *Ans.* 150 to 1.

18. The circumference of the earth is 21,600 nautical miles. What is the rate of motion per min. of a point situated on it, taking the degree at 69½ miles? *Ans.* 91,740 ft. per min.

19. A place on the earth moves at the rate of 45,870 ft. per min. What is its latitude? *Ans.* 60°.

20. A body moves at the rate of 3 miles in a quarter of an hour. Determine its velocity in ft. per sec.? *Ans.* 17⅗ ft.

STATICS.

21. Give examples of super-position of equilibrium.

22. If a line of $\frac{3}{8}$ in. represents a force of 7lbs., what force will a line of $2\frac{1}{8}$ in. represent? *Ans*. $39\frac{2}{3}$ lbs.

23. On the same scale, what must be the length of the line to denote 210 lbs.? *Ans*. $11\frac{1}{4}$ in.

24. A rope is pulled as follows; forward 15 lbs., backward 10 lbs.; forward 7 lbs., backward 9 lbs. What is the effect of the resultant force? *Ans*. forward 3 lbs.

25. Draw forces of 10 lbs., 3 lbs., and 7 lbs., so as to produce equilibrium by super-position of force.

26. Give examples of the transmissibility of force.

27. A vessel is moored in a river, by a bridle attached to two chains fastened to two buoys several yards apart. Show how to determine the ratio of the tension on the chains.

28. Two forces of 16 and 9 lbs. act at an angle of 60°. What is their resultant force? *Ans*. 21·932 lbs.

29. A weight of 9 tons is supported by two rafters, forming an angle of 90° What weight is borne by each rafter?

Ans. 6·36 tons nearly.

(*Note*. $R^2 = P^2 + P^2 = 2P^2 \therefore P^2 = \frac{R^2}{2} \quad \therefore P = \sqrt{\frac{R^2}{2}}$)

30. Two forces of 12 lbs. and 16 lbs. act at right angles to each other. What is their resultant? *Ans*. 20 lbs.

(*Note*. $R = \sqrt{\text{Sum of squares of forces.}}$)

31. The smaller the angle between two forces, the greater will be their resultant. Prove this?

32. Two forces of 8 and 6 lbs. respectively act at an angle of 30°. What is their resultant? *Ans*. 10·9 lbs. nearly.

33. Two boys pull a cart, the one with a force of 40 lbs., and the other with a force of 60 lbs. The cords are at an angle of 60°. What weight will they pull in addition to that of the cart, which weighs 30 lbs. neglecting friction? *Ans*. 57·3 lbs. nearly.

34. A force of 20 lbs. has two components equal to each other acting at an angle of 90°. What are they? *Ans*. 14·142 lbs.

35. A point in equilibrium is acted on by 3 forces. The first, of 21 lbs., acts at right angles to the second, of 16 lbs. What is the third force? *Ans.* 26.401 lbs.

36. Give the reason of the ascent of a kite.

37. A train weighing 60 tons, moving at a speed of 6 miles per hour, comes into collision with another weighing 130 tons, moving at a speed of 880 yards in 2′ 30″. What effect will the first train have in stopping the second?

38. A point in equilibrium has three forces acting upon it. Under what condition do they act, and in what directions?
Ans. In the directions of the sides of a triangle, taken in order.

39. A B is a force acting upon the point A. Resolve A into two components acting upon the same point.

40. A ship having to cross to a point on the opposite side of a channel down which a current runs, steers her course diagonally up the channel. Why is this?

41. Six equal forces acting upon a body keep it in equilibrium. At what angle are they inclined to each other? *Ans.* 60°.

42. Two forces acting together have a resultant of 10 lbs. and when acting in opposite directions their resultant is 2 lbs. What are the forces? *Ans.* 6 lbs. and 4 lbs.

43. A string supports a weight of 6 lbs. at one end, 7 lbs. 3 feet above the first, and 2 lbs. 2 feet above the second weight; what are the tensions on each part of the string? *Ans.* 2 lbs., 9 lbs., 15 lbs.

44. If in the last question the tension of the upper part be represented by a line $7\frac{1}{2}$ inches long, what length will represent the tensions of the other portions? *Ans.* $33\frac{3}{4}$ in., $56\frac{1}{4}$ in.

45. How does the rudder of a boat act?

46. Draw a force of 10 lbs., acting on a point, a second force of 5 lbs. acting on the same point at an angle of 45° to the first and a third force of $7\frac{1}{2}$ lbs. acting on the same point at an angle of 45° to the second.

47. A and B are concurrent parallel forces, $A = 14$, $B = 9$, and the distance between them is 26 ins. Where does the resultant act, and what is its magnitude?
Ans. Position, $10\frac{4}{23}$ ins. from A. Magnitude, 23.

48. A farmer harnessed a horse pulling 230 lbs., and a mule pulling 175 lbs., to the end of a yoke 3 ft. long attached to a cart, but found that they pulled the cart to one side. How must the draw-rope be fastened, to pull the cart in a straight line?

Ans. $20\frac{36}{81}$ ins. from the mule.

49. Two concurrent parallel forces of 30 and 24 lbs. act upon a rigid body. The smaller force acts at 4 ft. from the resultant. How far is the larger force from the resultant? *Ans.* 3·2 ft.

50. Two concurrent parallel forces of 20 lbs. and 112 lbs. are 33 ins. apart. How far is the smaller force from the resultant?

Ans. 2 ft. 4 ins.

51. A weight of 168 lbs. is carried by two men 5 ft. apart. The weight is suspended from a pole, and is 2 ft. from the first man. How much does each man carry? *Ans.* $100\frac{4}{5}$ lbs., and $67\frac{1}{5}$ lbs.

52. The resultant of two forces is 100 lbs. The smaller force is $\frac{7}{25}$ of the resultant, and is 4 ft. 3 in. from it. Find the other force and its distance from the resultant?

Ans. Force, 72 lbs.; distance, 1 ft. $7\frac{5}{6}$ in.

53. Why should the arms of a balance be equal?

54. Five weights, of 1, 2, 3, 4, and 5 lbs. respectively, are suspended at equal distances apart on a rod 40 ins. long. The rod is supported by a cord. At what distance from the end next the 1 lb. weight must it be attached so as to allow the rod to remain horizontal? *Ans.* $26\frac{2}{3}$ in.

55. A pole 20 ft. long rests with one end against a wall. 4 ft. from the wall a weight of 60 lbs. is suspended. Required the pressure on the other end of the pole, and that on the wall.

Ans. 12 lbs. on the other end of pole, and 48 lbs. on the wall.

56. A horizontal bar weighing 112 lbs. rests on two points, one under each end What is the pressure on each point?

Ans. 56 lbs.

57. A weight of 20 lbs. is suspended from a bar resting on two points. The weight is $9\frac{5}{8}$ in. from one end. What portion of the weight is borne by each point, the length of the bar being 6 ft. $4\frac{1}{2}$ in.?

Ans. $2\frac{1}{2}$ lbs. and $17\frac{1}{2}$ lbs.

58. The moment of a force 4 in. from a point = 20. How far must it be from the point to have a moment of 72? *Ans.* $14\frac{2}{5}$ in.

59. The earth revolves in an orbit concentric with that of Venus. When will the motion of Venus be most affected by the gravitation of the earth?

60. Prove that the C G of a circle is at the centre.

61. Why is it dangerous to stand up in a boat?

62. Two men carry a ladder weighing 60 kilogs. If the first man carries 12 kilogs., and the second 48 kilogs., and the ladder is 25 ft. long, where is the C G?

Ans. 5 ft. from the end next the second man.

63. Give an example of a body whose C G is not in the body.

64. A top when spinning remains upright. Why is this?

65. Two bodies, A and B, are floating 2 ft. from each other. A weighs 9 lbs. and B 4½ lbs. Where will they meet?

Ans. 8 ins. from A.

66. A cube is placed on a horizontal surface. To what angle can the surface be raised before the cube will roll? *Ans.* 45°.

67. A square table whose side = 48 ins., has weights of 4, 6, 8, and 10 lbs. respectively, placed on the corners. Where is the C G?

Ans. $\frac{4}{7}$ ft. from the centre, in the direction of the larger weights.

68. What forces act upon a body resting on an inclined plane?

Ans. Gravity, reaction, and friction.

69. Three equal bodies are placed in such positions as to be 4 ft. from each other. Where is the C G of the system?

70. A round table is supported on three legs, A, B, and C. A weight when placed on the edge of the table above B is supported, but if it is moved to the edge midway between B and C, it falls. Why is this?

71. Give examples of neutral equilibrium.

72. A man jumps 6 ft. on level ground. If he jumps on to a platform at a level from a railway train going at a speed of 40 mls. per hour, how far from the carriage will he alight, and why? *Ans.* 6 ft.

73. A slab of stone 4 ins. × 20 ins. × 1 in., whose specific gravity is 2·5, rests with one end near the edge of a square table, and its sides parallel to those of the table. On its further end a 1 lb. weight is placed. How far can the slab be pushed off the table without falling? *Ans.* 11·2218 ins.

74. Two bodies move with velocities of 6 and 42 ft. per sec. respectively, and their momenta are equal. What is the ratio between their weights? *Ans.* 7 to 1.

75. A power of 30 lbs. descending 2 ins. is employed to raise a weight through 3 ins. What is the weight? *Ans.* 20 lbs.

76. The weight ascends 4 ins. and the power descends 8 ins. The weight being 120 lbs., what is the power? *Ans.* 60 lbs.

77. A power of 10 balances a weight of 12. If P descends 16 ins., how far is W raised? *Ans.* 13$\frac{1}{3}$ ins.

78. Two men exert on a crane a force of 56 lbs. each to lift 8 tons. If 160 ins. of the fall are wound in in 2 secs., how long will it take to aise the weight 7 ft.? *Ans.* 2 mins. 48 secs.

THE LEVER.

79. What weight will balance 6 lbs. attached to the shorter arm of a lever 16 ins. long, the longer arm being 9 ins.? *Ans.* 4$\frac{2}{3}$ lbs.

80. 60 lbs. balances 40 lbs. at the extremities of a lever 15 ft. long. Where is the fulcrum? *Ans.* 7·2 ft. from the 60 lbs.

81. A substance weighs 196 grammes in one scale, and 121 in the other. What is its true weight? *Ans.* 154 grammes.

82. The fulcrum of a lever is 9 ft. from one end, and weights of 30 and 20 lbs. are suspended from the lever. When additional weights of 10 lbs. each are added to each end, where must the fulcrum be placed to keep up the equilibrium? *Ans.* 6·428 ft. from one end.

83. What power will raise 7000 lbs. with a lever 12 ft. long and the fulcrum at one end, W being 2 ft. from F? *Ans.* 1168$\frac{2}{3}$ lbs.

84. With the same lever as in the last question, what P will raise 8742 lbs., P being 9 ft. from F? *Ans.* 1942$\frac{2}{3}$ lbs.

85. A weight of 4 tons 11 cwt. 3 qr. 14 lbs. is to be raised by a lever 27 ft. long and a power of 343 lbs. Where is the fulcrum? *Ans.* 10$\frac{14}{31}$ ins. from the end.

86. The length of the long arm of a lever is 68$\frac{37}{40}$ ft., and the short arm is 3 ft., having a weight of 1 ton 4 cwt. 2 qrs. 13 lbs. attached to it. What weight on the long arm will balance it? *Ans.* 120 lbs.

87. The barrel of a capstan is 20 ins. diameter, and the paul rim 36 ins. Required the pressure on the pauls (Fig. 43) when a strain of 5 cwt. 4 lbs. is on the rope? *Ans.* $313\frac{1}{3}$ lbs.

88. Prove that the graduations on a steelyard are equal.

89. A beam 7 ft. long rests on two points under each end. 1 ft 5 ins. from one end is a weight of 4 cwt. 3 qrs. 21 lbs. What is the pressure on each of the points? *Ans.* $441\frac{7}{84}$ lbs., and $111\frac{77}{84}$ lbs.

90. A truck has 2 sets of wheels: one 6 ft. in diameter, and the other 4 ft. With which set will it be easiest to draw the truck over a rough road, and why?

91. The weight of a lever of the third order being neglected; when P = 224 lbs., P F = 2 ins., F W = 13 ins., what is W? *Ans.* 34·46 lbs.

92. A wheelbarrow containing 90 lbs. has handles 6 ft. long. The C G of the barrow and load is 2 ft. from the axle. If the barrow weighs 30 lbs., what power will lift it? *Ans.* 40 lbs.

93. A bent lever (Fig. 39), whose arms are 16 and 3 ins. long, has a weight of 20 lbs. attached to the shorter arm. What is the power? *Ans.* 3·75 lbs.

94. Two horses are attached to the levers of a capstan, and exert a force of 840 lbs. each. What load will they raise if the levers are 4 yds. long, and the axle is 3 ft. in diameter? *Ans.* 6 tons.

THE WHEEL AND AXLE.

95. A wheel has a radius of 1 ft., and the axle of $2\frac{1}{2}$ ins. What weight will be raised by a power of 127 lbs.? *Ans.* $609\frac{3}{5}$ lbs.

96. A wheel has a radius of 2 ft., and a power of 80 lbs. is acting upon it, and balancing 1 cwt. 8 lbs. at the axle. What is the radius of the axle? *Ans.* 1·4 in.

97. 7 lbs. sustains 164 lbs., on an axle 25·1320 in. in circumference. What is the radius of the wheel? *Ans.* 7 ft. $9\frac{5}{7}$ ins.

98. 700 lbs. is raised by a power of 90 lbs. applied to a wheel of 3 ft. radius. What is the radius of the axle? *Ans.* 4·628 in.

99. If P descends 6 ft. while W ascends 3 ins., what is the ratio between the wheel and axle? *Ans.* 24 : 1.

100. A winch 18 ins. long is driven by a power of 90 lbs. The larger part of the axle is 6 ins. in diameter and the smaller 4 in. What weight will be lifted? *Ans.* 3240 lbs.

101. The effective length of a winch is 1 ft., and a power of 70 lbs. acts upon it. The larger axle is 8 ins. in diameter. What is the radius of the smaller when 15 cwt. is balanced by the power? *Ans.* 3 ins.

102. Four friction wheels have a mechanical advantage of 4 to 1 each; what weight on the axle of the fourth wheel will 2 lbs. balance? *Ans.* 512 lbs.

103. Five drums, A, B, C, D, and E, are connected by bands; A revolves from left to right, and is connected by *crossed* bands with B, which is connected by *uncrossed* bands with C, which is connected by *crossed* bands with D, which is connected by *uncrossed* bands with E. If the drums are of such sizes that B revolves 3 times to one revolution of A, and so on, how many times will E revolve in 4 turns of A, and in what direction? *Ans.* From left to right, 324 times.

104. A power of 80 lbs. raises 17 cwt. 3 qrs. 2 lbs., when applied to a pinion having 12 teeth. How many teeth are there in the large wheel? *Ans.* 300.

105. A wheel 14 ins. in diameter has a ratchet wheel on it, and the axle 3 ins. in diameter supports 75 lbs. What is the pressure on the paul? *Ans.* $16\frac{1}{14}$ lbs.

106. A force of 27 lbs. is applied to a winch 1 ft. 4 ins. long; the pinion has 8 teeth, the wheel 112, and the barrel is 8 ins. diameter. What weight will be raised? *Ans.* 1512 lbs.

107. The wheel has 64 teeth, the pinion 4 teeth, the radius of the barrel is 2 ins., and the length of the winch is 18 ins. What force will raise 2736 lbs.? *Ans.* 19 lbs.

108. A power of $22\frac{7}{15}$ lbs. will raise 674 lbs. The barrel is 3 ins. in radius, and the winch is 1 ft. 3 ins. long. What are the relative velocities of the wheels? *Ans.* 6 to 1.

109. Two men exert forces of 248 and 200 lbs. respectively on an axle to which two levers are attached, the one being 4 ft. and the other 5 ft. long. If the axle has a radius of 10 ins. what weight will be raised? *Ans.* 2390·4 lbs.

THE PULLEY.

110. A gun-tackle whose fall is pulled by 7 men, each exerting a force of 96 lbs., is fixed to the fall of a luff-tackle, the upper block being fixed. What weight will be lifted? *Ans.* 2 tons 14 cwt.

111. In the last question what weight would be lifted if the lower block were immovable? *Ans.* 3 tons 12 cwt.

112. The following purchase is rigged. A rope is rove through a fixed block, and one end fixed. The other end has a twofold purchase attached to it, to the fall of which a gun-tackle is hooked, and the hooks of the lower twofold purchase block and the lower gun-tackle block are hooked to the weight. If 212 lbs. is applied to the gun-tackle fall, what weight will be lifted, disregarding friction?
Ans. 1 ton 6 cwt. 2 qrs.

113. In the preceding question what strain is there on the strap of the single block end, and what weight will the fixed end of the whip sustain?
Ans. 2 tons 17 cwt. 3 qrs. 4 lbs., on the single block end, and 1 on 8 cwt. 3 qrs. 16 lbs. on fixed end of whip.

114. What power will support 1222 lbs. with 3 movable blocks, weighing 14, 10, and 4 lbs. respectively, and 3 cords whose ends are attached to a beam? *Ans.* 159 lbs.
(*Note.* P = W + weight of block ÷ 2; and so on with each block.)

115. How much of the fall of a luff-tackle must be hauled in to raise 116 lbs. 2 ft. when the upper block is fast to the fall of a whip and runner? *Ans.* 16 ft.

116. With a system as in Ex. 114., what weight will 126 lbs. support when the blocks weigh 7 lbs., 11 lbs., and 17 lbs.?
Ans. 941 lbs.

117. What tackle must I use to raise half a ton by means of 373⅓ lbs.? *Ans.* A gun-tackle upper block to the weight.

118. If a tackle is rove so that 4 lbs. will lift 72 lbs., and a man pulls 3 ft. at each effort, how many efforts must he make to lift the weight 7 ft.? *Ans.* 42.

119. A whip is fastened to the fall of a gun-tackle, and is pulled with a force of 5 lbs. What force will be exerted on the upper movable block? *Ans.* 30 lbs.

120. In pulley systems it is advantageous to have sheaves as large as is convenient. Why is this?

THE INCLINED PLANE.

121. If 3 tons rest on an inclined plane 24 ft. long and 2 ft. high, what is the ratio between the force down the plane and the weight? *Ans.* 1 : 12.

122. If a weight of 2 tons 14 cwt. rest upon a plane whose inclination is 1 in 27, what is the force down the plane? *Ans.* 224 lbs.

123. A weight of 300 lbs. has to be raised 14 ft. by a plane 36 ft. long; what force will be required? *Ans.* $117\frac{1}{3}$ lbs.

124. A power of 150 lbs. is exerted along a plane 15 ft. long and 3 ft. high. What weight will it raise? *Ans.* 750 lbs

125. A board rests on a cart 3 ft. high, and its lower edge is 14 ft from the cart. What weight will a power of 58 lbs. lift when applied parallel to the ground? *Ans.* $270\frac{2}{3}$ lbs.

126. A power of 1 ton 12 cwt. 3 qrs. 22 lbs. is exerted over a plane. 123 ft. long, and 2 ft. 6 ins. high. What weight will it raise, the power being parallel to the plane? *Ans.* 81 tons 108 lbs.

127. A force of 27 lbs. is applied parallel to the horizon to raise a weight to the top of a wall 4 ft. high from a point 30 ft. distant. What is the weight? *Ans.* $202\frac{1}{2}$ lbs.

128. A power of 20 lbs. will raise 600 lbs. 4 ft. high; what is the length of the plane when the force is parallel to it? *Ans.* 120 ft.

129. 10 cwt. 3 qrs. 15 lbs. is exerted parallel to a plane 114 ft. 3 ins. long, and 1 ft. 9 ins. high. What weight will it raise? *Ans.* 71 tons 5 cwt. 2 qrs. 21 lbs.

130. A weight of 1600 lbs. has to be raised 20 ft. by a plane 104 ft. long, the power acting parallel to the plane; what is the power? *Ans.* $307\frac{9}{13}$ lbs.

131. 35 lbs. applied along a plane will raise 7000 lbs. 15 ft. what is the length of the plane? *Ans.* 1000 yds.

132. 1 ton 3 cwt. 2 qrs. 21 lbs. is to be raised 17 ft., by a plane 207 ft. long, the power acting parallel to the plane. What must be the power? *Ans.* $217\frac{179}{207}$ lbs.

133. A weight of 50 tons is supported on a plane sloping 1 in 50, by a rope and 6 horses. How much must each horse pull? *Ans.* $373\frac{1}{3}$ lbs.

134. A force of 2 cwt. 1 qr. 13 lbs. is exerted along a plane 12 feet long and 2 feet high. What weight will be raised?

Ans. 14 cwt. 22 lbs.

135. 1200 lbs. is supported by two forces of 5 cwt. 1 qr. 12 lbs. each, one acting along the plane, and the other parallel to the base. What is the ratio of the length of the plane to the height?

Ans. 5 to 4.

136. 1 ton 14 cwt. 1 qr. 16 lbs. is to be raised 6 feet by a power parallel to the horizon, the base of the plane being 72 feet long. What is the power? *Ans.* 321 lbs.

137. 28 lbs. will raise 190 lbs. 4 feet. What is the length of the plane, the power acting along it? *Ans.* $27\frac{1}{7}$ ft.

138. A force of 180 lbs. is applied parallel to the horizon, to raise 976 lbs. to a platform 3 ft. high. How far must the lower edges of the planks forming the plane be from the foot of the platform? *Ans.* $16\frac{4}{15}$ ft.

139. If the power acts parallel to the plane, and supports the weight, what must be the angle of the plane? *Ans.* 60°

THE WEDGE.

140. A wedge 15 ins. long and 2 ins. thick, is driven with a force of 116 lbs. What weights will it separate? *Ans.* 1740 lbs.

141. A stone weighing 5 tons, having been sawn into halves, is required to have a 6 in. iron bar inserted in the cut. If a wedge 10 ins. long and 6 ins. thick is used to separate the halves, what force must be applied to it? *Ans.* 15 cwt.

142. A power of 120 lbs. separated 2 bodies of 700 lbs. each with a wedge 12 ins. long. What was its thickness? *Ans.* $2\frac{4}{70}$ ins.

143. A vessel is raised by 300 wedges, each 12 ins. long, and $1\frac{1}{2}$ in. thick, and a force of 130 lbs. on each wedge; what is the weight of the ship? *Ans.* 137 tons 1 cwt. 8 lbs.

144. A ship of 2000 tons was raised by 500 wedges of 46 ins. long and 1 in. thick. What force was applied to each? *Ans.* $194\frac{18}{23}$ lbs.

THE SCREW.

145. A screw 1 ft. in circumference and ½ in. pitch has a power of 57 lbs. applied to it. What weight will it raise? *Ans.* 1,368 lbs.

146. A screw of 1 in. pitch has a power of 5000 lbs. applied to it, which raises 80,000 lbs. What is the circumference of the screw? *Ans.* 16 ins.

147. A screw of 3½ ins. radius has a power of 1¼ lbs. applied to it. What weight will be raised, supposing the pitch is ¼ in.? *Ans.* 109·9950 lbs.

148. In a differential screw, one screw of which has 5 threads in an inch and the other 4, what weight will 65 lbs. applied to it raise, the mean diameters of the screws being 3 ins.? *Ans.* 12,252·22 lbs.

149. A screw is worked by means of a lever 2 ft. 6 ins. long, and the pitch is ½ in. What weight will a man raise if he exerts a force of 50 lbs. on the lever? *Ans.* 1570·8 lbs.

150. Why is Hunter's screw admirably adapted for minute movements?

151. A screw advances 6 in. in 48 revolutions, and a power of 50 lbs. is applied to a lever 17 ins. long. What weight is the screw capable of raising? *Ans.* 2136·08 lbs.

152. A screw advances 7 ins. in 5½ turns, and the resistance to be moved is 1000 lbs. When the lever is 1 ft. 6 ins. long, what must be the power? *Ans.* 225 lbs.

153. Show how the efficacy of a screw depends upon the angle of its pitch.

154. What power will raise 1432 lbs., when the pitch of the screw is $\frac{1}{11}$ in. and the circumference described by the power is 2 ft.? *Ans.* 13 lbs.

STATICS.

MISCELLANEOUS.

155. Give an example of each of the modifications of motion mentioned in Chapter XII.

156. A steam engine has a stroke of 4 ft., and the axle has a drum fixed on it of 1 ft. 4 ins. in radius. If a power of 1 ton is applied to the piston, what is the ratio of the force on the crank-pin and at the circumference of the drum? *Ans.* 3 to 2.

157. The crank of a steam engine is 9 ins. long, and the engine makes 100 revolutions in a min. What is the speed of the piston? *Ans.* 300 feet per min.

158. An engine of 2 ft. stroke has a speed on the piston of 360 ft. per min. The throw of the excentric is 4 ins. What is the speed of its connecting-rod? *Ans.* 120 ft. per min.

159. One end of a ladder rests upon a road, and the other against a vertical wall. What are the forces acting upon the ladder?

160. Why could not a horse draw a load if there were no friction?

161. A body will remain at rest upon a slope of 45°. What is the co-efficient of friction?

162. A truck on a railway weighs 4480 lbs., and a force of 1680 lbs. applied to it horizontally will just move it; what is the co-efficient of friction? *Ans.* $\frac{3}{8}$.

163. A ton weight has to be drawn in a cart. Would it be more economical of force to have large or small wheels to the cart?

164. A block of oak weighing 50 lbs. is drawn along the surface of an elm plank. What force must be used to draw it? *Ans.* 19 lbs.

165. An iron box weighing 1 cwt. is dragged over an oak plank, what force is exerted? *Ans.* 56 lbs.

166. A ladder 15 yds. long, whose C.G. is 25 ft. from its lower end rests against a wall, the foot being on a horizontal plane. If the co-efficient of friction between the ladder and the plane is $\frac{5}{12}$, how far can the foot be drawn from the wall without the ladder sliding? *Ans.* 27 ft,

167. A weight of 1250 lbs. has to be raised by a 4 in. dry rope, and a sheave 4·36 ins. radius. What is the resistance due to the stiffness of the rope? *Ans.* 175·275 lbs.

168. The volume of the planet Saturn is 697·7 that of the earth, but his sp. grav. is only 0·13. Compare his mass with that of the earth. *Ans.* 90701 to 1.

169. Two inelastic bodies, whose weights are 14 and 21 lbs. respectively, are made to impinge on one another. The velocity of the smaller body is 60 ft. per sec., and it destroys the velocity of the larger one. What is the velocity of the latter? *Ans.* 40 ft. per sec.

170. A shot weighing 40 lbs. is fired from a gun weighing 4 tons, and leaves the gun with a velocity of 100 ft. per sec. What is the recoil of the gun? *Ans.* ·0446429 ft. per sec.

KINETICS.

171. A body had at one sec. a velocity of 16 ft. per sec., and 4 secs. later a velocity of 36 ft. per sec. What is the acceleration? *Ans.* 5 ft.

172. A body starts from rest, and in 7 secs. has a velocity of 56 ft. per sec. What is the acceleration? *Ans.* 8 ft.

173. A body starting from rest, moves for 7 secs. with an acceleration of 540 ft. per minute. What is the velocity at the end of the 7th sec.? *Ans.* 63 ft. per sec.

174. The acceleration of a body is 20 ft. How far will it be at the end of the 4th sec.? *Ans.* 200 ft.

175. A body falls with a velocity of 256 ft. per sec. How long has it been falling? *Ans.* 8 secs.

176. A body is thrown upwards to a height of 156·25 feet, which it reaches in 3·125 secs. What was its initial velocity? *Ans.* 100 ft. per sec.

177. A body is thrown downward with a velocity of 300 ft. per minute. What velocity will it have when it has fallen 3 secs.? *Ans.* 396 ft. per min.

178. When will a body falling with g have a velocity of 320 ft. per sec.? *Ans.* in 10 secs.

179. When would the body in the last example have acquired a velocity of 256 ft. per sec.? *Ans.* in 9¼ secs. from its start.

180. A body is thrown upwards with a velocity of 48 ft. per sec. What velocity will it have at the end of 3 seconds? *Ans.* None.

181. Gravity at the sun's surface is 27·1, the earth's being unity. What is the equivalent of a cwt. on his surface? *Ans.* 3035·2 lbs.

182. If a meteorolite falls to the earth from a height of 100 miles, with what velocity does it strike the ground?
Ans. 16,187·5 ft. per sec.

183. What is g at the sun's surface? *Ans.* 867·2 ft. per sec.

184. What velocity would be acquired by a body falling 4 seconds at the sun's surface? *Ans.* 3468·8 ft. per sec.

185. A body is thrown upwards with a velocity of 1920 yds. per min. In what direction and with what velocity will it be moving at the end of the 4th sec.? *Ans.* Downwards at 32 ft. per sec.

186. A stone falls to the bottom of a cliff in 6 sec. What is the height of the cliff? *Ans.* 576 ft.

187. A stone is thrown downwards with a velocity of 5 ft. per sec. What will be its velocity at the end of the 11th sec.?
Ans. 407 ft. per sec.

188. A stone from the edge of a cliff 784 ft. high reaches the foot in 7 sec. What was the acceleration? *Ans.* g.

189. An 8 lb. stone falling from a cliff reached the ground with a momentum of 2560 lbs. What was the height of the cliff?
Ans. 1600 ft.

190. A boy throws a stone downwards with a velocity of 280 ft. per sec., and at the same instant another boy throws a stone upwards with equal force. What is the ratio of the two velocities at the end of the 6th sec.? *Ans.* 59 to 10.

191. What force would be necessary to generate a velocity of 35 ft. per sec. at the end of 3 secs.? *Ans.* $11\frac{2}{3}$ ft.

192. A stone is thrown upwards with a velocity of 120 ft. per sec. When will it reach the ground? *Ans.* In $7\frac{1}{2}$ secs.

193. I throw a stone from a certain point with a velocity of 12 ft. per sec. How far will it descend in 6 secs.? *Ans.* 648 ft.

194. A stone 4 lbs. weight is dropped from a cliff 1600 ft. high. With what force will it strike the ground? *Ans.* 1280 lbs.

195. A stone, S, is dropped from the top of a cliff, and 1 sec. after another stone, T, is dropped from a point 150 ft. below. When will S overtake T? *Ans.* In $5\frac{3}{16}$ secs.

196. A body is thrown downwards with a velocity of 7ft. per sec. What is its velocity at the end of 9 secs.? *Ans.* 335 ft. per sec.

197. A body ascends $3\frac{1}{11}$ secs. What was its initial velocity? *Ans.* 100 ft. per sec.

198. A pendulum in Spitzbergen is required to beat 3 secs. What must be its length? *Ans.* 352·9296 in.

199. What is the rate of oscillation of a pendulum in London whose length is 12 feet? *Ans.* 1·923 secs.

200. A shot is fired with a velocity of 900 ft. per sec. at an angle of 45°. To what height will it rise? *Ans.* 3164·625 ft.

201. If a body projected from a gun has a range of 3750 ft., and a horizontal velocity of 600 ft. per sec., what is its vertical velocity? *Ans.* 100 ft. per sec.

202. The counterpoise of the crank on a fly-wheel weighs 60 lbs. The wheel is 37·6980 feet in circumference, and makes 30 revolutions per min. What is the centrifugal force of the balance? *Ans.* 642·71712 lbs.

203. A nonelastic body, A, weighing 7 lbs., moving with a velocity of 60 ft. per sec., overtakes a similar body, B, which weighs 16 lbs., and moves at the rate of 11 ft. per sec. What is the common velocity after impact? *Ans.* $25\frac{21}{23}$ ft. per sec.

204. A body, A, weighing 8 lbs. moving at the rate of 20 ft. per sec. meets another body, B, weighing 6 lbs. moving at the rate of 12 ft. per sec. What is the common velocity after impact? *Ans.* $6\frac{2}{7}$ ft.

205. The engine of a train moving 4 ft. per sec. weighs 30 tons. It is moving round a curve with a radius of 400 yds. What is the outward pressure on the rails? *Ans.* 28 lbs.

206. Two unequal bodies are fixed at the ends of a rod, whose weight can be neglected, and are describing circles with uniform velocities round a common centre. At what point of the rod is the axis of the system? *Ans.* At the C.G. of the two bodies.

207. Two nonelastic bodies moving in the same direction come into collision, A weighs 7 lbs. and moves at the rate of 60 ft. per sec. and B weighs 16 lbs. and moves with a speed of 11 ft. per sec. What is the common velocity after impact? *Ans.* $25\frac{21}{23}$ ft. per sec.

208. In the above case, what velocity is lost by A and what gained by B?

Ans. $18\frac{4}{23}$ ft. per sec. lost by A; $7\frac{11}{23}$ ft. per sec. gained by B.

209. Two inelastic bodies, A of 12 lbs. with a velocity of 16 ft., and B of 8 lbs. and a velocity of 21 ft., meet. Determine the common velocity after impact? *Ans.* 1·20 ft.

210. How much will a bar of cast-iron 1 in. square and 80 ft. long be stretched by a force of 2 tons? *Ans.* ·00210 ft.

211. A bricklayer who weighs 145 lbs. carries a hod weighing 15 lbs., containing 20 bricks, each weighing 6 lbs. up a ladder 40 ft. high. If he can do 3,537,600 units of work per day how many bricks will he carry up the ladder in a day? *Ans.* 4020.

212. A machine is so constructed as to enable a man to do 2,168,320 units of work per day of 8 hours. The machine is capable of raising two tons. How long will it take to raise that weight 100 ft.? *Ans.* 99·128 min.

213. How many gallons of water will a 400 horse-power engine pump up from a depth of 400 yds. in an hour, supposing a gallon of water to weigh 10 lbs.? *Ans.* $138\frac{1}{3}$ galls.

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